Georgia Institute of Technology
School of Mathematics

Second practice exam for MATH 2401, Sections J1 and J2

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No books, notes or calculators of any kind are allowed. Maximum number of points is 40 (10 for each problem). Duration of test is 50 min.

Justify all your answers

1. Let $f(x, y, z) = xyz$ and $z = r\cos\phi$, $x = r\sin\phi\cos\theta$, $y = r\sin\phi\sin\theta$. Find the partial derivatives of f with respect to r, ϕ and θ.
2. Find the absolute maximum and minimum of the function $x^2 - 2y^2$ in the rectangle

$$1 \le x \le 2, \quad 3 \le y \le 4.$$

3. (a) Find a normal vector to the surface $x^2 + 4y^2 = z^2$ at its point $(3, 2, 5)$.
(b) Find the equation of the tangent plane at that point.
(c) Find a parametrization of the normal line at that point.
4. A rectangular box has three of its faces on the coordinate planes and one vertex in the first octant on the paraboloid $z = 4 - x^2 - y^2$. Determine the maximum volume of the box.