BUCHI'S EQUATION AND AN APPLICATION OF SOME CONJECTURES OF LANG AND BOMBIERI

ABSTRACT. Buchi's equation for the exponent 2 and M variables is the system $(x_{n+2}^2 + x_{n+1}^2 = 2x_n^2 + 2)$ for $n = 0, ..., M$. It is satisfied by all successive integers: $x_n = n + k$ with k an arbitrary constant. For $M < 4$ the system has solutions. For $M = 5$ no rational solutions have been found by experimentation. We ask:

Question: Is it true that the system has no nontrivial (non-successive) integer or rational solutions for some large enough M?

We will give a short account of the status of the problem (it is currently open but a positive answer follows from a conjecture of Lang and a "question" of Bombieri, the result is due to Vojta). The analogue of the problem has a positive answer in global fields of positive characteristic and in fields of global meromorphic functions - usual and p-adic (via Nevannlina Theory). We will also explain how the problem came from an effort to strengthen the negative answer to "Hilbert's tenth problem".