2nd Homework

Due Fri, Feb 7, in class.

1. Evaluate
$$S_1 = \sum_{k=0}^{m} \binom{n-k}{m-k}.$$
HINT: $S_1 = [x^{n-m}] \sum_{k=0}^{m} (1+x)^{n-k}$.

2. Evaluate
$$S_2 = \sum_{k=0}^{m} (-1)^k \binom{n}{k} \binom{n}{m-k}.$$
HINT: $S_2 = [x^m] \left((1-x)^n (1+x)^n \right)$.

3. Evaluate
$$S_3 = \sum_{k=m}^{n} \binom{n}{k} \binom{k}{m}.$$

4. Evaluate
$$S_4 = \sum_{k=0}^{m} (-1)^k \binom{n}{k}.$$
HINT: Use $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$.

5. Let $[n] = \{1, \ldots, n\}$, $X \subset [n]^k$. Call X *good* if any two elements of it do <u>not</u> agree on any of their k coordinates. Let $x \in [n]^k$. In how many good sets does x belong?

6. In how many ways can we go up n stairs if we can take on or two steps at a time?

7. §3.13, problem 21.

Note: The notation $[x^k]f$ means, as explained in class, "the coefficient of x^k in the expression f."