University of Illinois at Urbana–Champaign
Department of Mathematics

First Midterm Exam for **MATH-315** (Linear Algebra), Section E2
Instructor: M.N. Kolountzakis

February 24, 1997

YOUR NAME:

SCORE:

1	
2	
3	
4	
5	
TOTAL	

No books, notes or calculators allowed
Maximum number of points: 50
Duration: 1 hour

*** Explain all your answers ****

GOOD LUCK!

1 (a) (5 points) Describe the set of all solutions of the following linear system by reducing its augmented matrix to **row echelon** form:

$$\left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 2 & 2 & 3 \\ 1 & 1 & 1 & 2 & 3 & 2 \end{array}\right).$$

(b) (5 points) Find the inverse of the following matrix without using determinants

$$\begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & -2 & -3 \end{pmatrix}.$$

2 (10 points) You are reminded that, if A is a $n \times n$ matrix, then the $n \times n$ matrix adj A satisfies

$$A \cdot \operatorname{adj} A = (\det A) \cdot I.$$

Show that

$$\det(\operatorname{adj} A) = (\det A)^{n-1}.$$

3 (2 points each) Consider the following subsets of $\mathbf{R}^4$. Which are subspaces of $\mathbf{R}^4$?

The vectors $\mathbf{x} = (x_1, x_2, x_3, x_4)^\top$ such that

1. $x_1 - x_2 = 0$,
2. $|x_1| - |x_2| = 0$,
3. $x_1 + x_2 + x_3 + x_4 = 0$,
4. $x_1 + x_2 = 0$ and $x_3 + x_4 \neq 0$,
5. $x_1^2 + x_2^2 = 2x_1x_2$.

Your answers should be justified. You should check whether the set in question is closed under addition and multiplication by scalars. If yes, you should prove that, if not, an example suffices to show that they are not closed.

4 (10 points) Let $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ be a spanning set for the vector space V (this means that every vector in V can be written as a linear combination of $\mathbf{v}_1, \ldots, \mathbf{v}_n$). Let $\mathbf{v}$ be *any* vector of V. Show that the collection of vectors

$$\mathbf{v}, \mathbf{v}_1, \ldots, \mathbf{v}_n \tag{1}$$

is linearly dependent.

HINT: You should exhibit a linear combination of the vectors in (1) which is equal to the vector $\mathbf{0}$ and not all its coefficients are equal to 0.

5 (10 points) Show that the collection of vectors in $\mathbf{R}^3$

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \mathbf{v}_2 = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \mathbf{v}_3 = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$$

forms a basis of $\mathbf{R}^3$. Express the vector $\mathbf{w} = (3, -1, 4)^\top$ as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

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