University of Illinois at Urbana–Champaign
Department of Mathematics

Second Midterm Exam for **MATH-315** (Linear Algebra), Section E2
Instructor: M.N. Kolountzakis

April 18, 1997

YOUR NAME:

SCORE:

1	
2	
3	
4	
5	
TOTAL	

No books, notes or calculators allowed
Maximum number of points: 50
Duration: 50 minutes

*** Explain all your answers ****

GOOD LUCK!

1 (10 points) Let

$$\begin{aligned}\mathcal{V} &= [\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3] = [(1,1,1)^\top, (2,3,2)^\top, (1,5,4)^\top] \\ \mathcal{U} &= [\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3] = [(1,1,0)^\top, (1,2,0)^\top, (1,2,1)^\top],\end{aligned}$$

be two ordered bases of $\mathbf{R}^3$.

Find the transition matrix from $\mathcal{V}$ to $\mathcal{U}$ and use this matrix to find the coordinates of $\mathbf{x} = 3\mathbf{v}_1 + 2\mathbf{v}_2 - \mathbf{v}_3$ with respect to the basis $\mathcal{U}$.

2 (10 points) Let

$$A = \begin{pmatrix} 1 & -2 & 1 & 1 & 2 \\ -1 & 3 & 0 & 2 & -2 \\ 0 & 1 & 1 & 3 & 4 \\ 1 & 2 & 5 & 13 & 5 \end{pmatrix}.$$

Find a basis for the row-space of A. What is $\operatorname{rank}(A)$?

3 (10 points) Let $A \in \mathbf{R}^{m \times n}$, $B \in \mathbf{R}^{m \times m}$, and assume that B is non-singular.

Show that $N(A) = N(BA)$.

4 (10 points) Let $L : \mathbf{R}^2 \to \mathbf{R}^3$ be the linear transformation defined by

$$L(\mathbf{x}) = (x_2, x_1 + x_2, x_1 - x_2)^\top.$$

Find the matrix representation of L with respect to the ordered bases

$$\mathcal{U} = [\mathbf{u}_1, \mathbf{u}_2] = [(1,2)^\top, (3,1)^\top]$$

and

$$\mathcal{V} = [\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3] = [(1,0,0)^\top, (1,1,0)^\top, (1,1,1)^\top].$$

5 (10 points) Let V be an inner-product space and $\mathbf{x}$ and $\mathbf{y}$ be two orthogonal elements of V.

Show the Pythagorean theorem:

$$\|\mathbf{x} - \mathbf{y}\|^2 = \|\mathbf{x}\|^2 + \|\mathbf{y}\|^2.$$

HINT: For any $\mathbf{v} \in V$ we have $\|\mathbf{v}\| = \sqrt{\langle \mathbf{v}, \mathbf{v} \rangle}$.

SCRATCH PAGE