Georgia Institute of Technology
School of Mathematics

First practice exam for MATH 2401, Sections J1 and J2

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No books, notes or calculators of any kind are allowed. Maximum number of points is 40 (10 for each problem). Duration of test is 50 min.

Justify all your answers

1. A particle is at position $\vec{r}(t) = (\cos t, t, \sin t)$ at time t. What is the length traversed by that particle from time $t = 1$ to time $t = 4$?
2. For the curve given parametrically by $\vec{r}(t) = (2t, t^2, t^3/3)$:
(a) Find the curvature κ at time t.
(b) Determine the tangential and normal components, $\vec{a}_T$ and $\vec{a}_N$ of the acceleration at time t.
3. A particle of mass $m = 5$ moves under the force $\vec{F}(t) = (-5\cos t, -5\sin t, 0)$. At time $t = 0$ the particle is at $(1, 0, 0)$ and has velocity vector $(0, 1, 1)$. After how much time will the particle have covered length equal to 10 (since time $t = 0$)?
4. Find the maximum domain of the function $f(x, y) = \sqrt{1 - x^2} + \sqrt{2 + y^2}$ and find its interior and boundary points. Compute the partial derivatives of f.