

Formal duality in finite cyclic groups

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Numerical computations by Cohn, Kumar, and Schürmann in energy minimizing periodic configurations of density ρ and $1/\rho$ revealed an impressive kind of symmetry, the so-called *formal duality*. Formal dual subsets of \mathbb{R}^n satisfy a generalized version of Poisson summation formula, and it is conjectured that the only periodic subsets of \mathbb{R} of density 1 possessing a formal dual subset, are \mathbb{Z} and $2\mathbb{Z} \cup (2\mathbb{Z} + \frac{1}{2})$.

In this talk, we will present recent progress on this conjecture by the speaker, utilizing several tools from different areas of mathematics, such as (a) the field descent method by Bernhard Schmidt, used chiefly on the circulant Hadamard matrix conjecture, (b) the “polynomial method” by Coven & Meyerowitz used in the characterization of sets tiling \mathbb{Z} or \mathbb{Z}_N , and (c) basic arithmetic of cyclotomic fields.