# Convex Geometric Analysis

**Workshop on the occasion of the retirement of**
**Professor Souzana Papadopoulou**

**Department of Mathematics**
**University of Crete**
**September 10-13, 2012**

## Talks

The workshop will take place in the amphitheater "Maria Manassaki" located on the ground floor of the Students' Center at the campus of the University of Crete in Voutes.

### Monday, September 10, 2012

09:00-09:30 – *Registration*

09:30-10:20 – **Rolf Schneider**, *Canonical diametric completions in normed spaces.*

10:30-10:50 – *Coffee Break*

10:50-11:40 – **Alain Pajor**, *Weak and strong moments estimates for convex measures.*

11:50-12:40 – **Emanuel Milman**, *A generalization of Caffarelli's Contraction Theorem via (reverse) Heat-Flow.*

12:50-14:10 – *Lunch Break and Coffee*

14:10-15:00 – **Beatrice-Helen Vritsiou**, *A proof of the Bourgain-Milman inequality via the isotropic position.*

15:10-16:00 – **Alexander Koldobsky**, *Stability and separation in volume comparison problems.*

16:10-17:00 – **Aljoša Volčič**, *Iterations of Steiner symmetrizations.*

### Tuesday, September 11, 2012

09:30-10:20 – **Spiros Argyros**, *Saturated under constraints norms and the Hereditary Invariant Subspace Property.*

10:30-10:50 – *Coffee Break*

10:50-11:40 – **Vitali Milman**, *The reasons behind some classical constructions in analysis.*

11:50-12:40 – **Peter Gruber**, *Normal bundle cones of convex bodies.*

12:50-14:10 – *Lunch Break and Coffee*

14:10-15:00 – **Achim Clausing**, *Pòlya operators and convexity.*

15:10-15:50 – **Apostolos Giannopoulos**, *$M$ and $M^*$-estimates for centrally symmetric convex bodies.*

15:50-16:30 – **Mihalis Kolountzakis**, *In which domains can one do Fourier Analysis?*

16:30-17:00 – *Coffee Break*

17:00-18:00 – *Retirement Celebration Honoring Professor Souzana Papadopoulou*

20:30 – *Workshop Dinner*
Please register for the dinner on Monday. Cost for non-speakers is 25Eur.

**Wednesday, September 12, 2012**

09:30-10:20 – **Mathieu Meyer**, *An application of shadow systems to Mahler's Conjecture.*

10:30-11:20 – **Antonis Tsolomitis**, *Random polytopes in convex bodies.*

11:30-11:50 – *Coffee Break*

11:50-12:40 – **Menelaos Karavelas**, *Exact bounds on the number of faces of the Minkowski sum of convex polytopes.*

12:50-13:40 – **Lefteris Markessinis**, *Classical positions of centrally symmetric convex bodies.*

**Thursday, September 13, 2012**

09:30-10:20 – **Martin Henk**, *Successive minima inequalities.*

10:30-11:20 – **Alexander Litvak**, *Vertex index of symmetric convex bodies.*

11:30-11:50 – *Coffee Break*

11:50-12:40 – **Gergely Ambrus**, *Polarisation problems.*

12:50-13:40 – **Michel Marias**, *Spectral multipliers on locally symmetric spaces.*

## Abstracts

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**Gergely Ambrus**, Renyi Institute, Hungary.

*Polarisation problems*

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We prove several results related to the following problem. Given $n$ unit vectors $u_1, \ldots, u_n$ in a real Hilbert space $H$, there always exists another unit vector $v$ of $H$, which satisfies

$$\sum_{i=1}^{n} \langle u_i, v \rangle^{-2} \leq n^2.$$

This conjecture generalises the so-called complex plank theorem of K. Ball to the real setting. We are also going to describe the connections to homogeneous polynomials, inverse eigenvectors, and John's ellipsoid theorem.

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**Spiros Argyros**, National Technical University of Athens, Greece.

*Saturated under constraints norms and the Hereditary Invariant Subspace Property*

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The invariant subspace problem is a well known and central problem of the operator theory on Hilbert space and naturally stated for every separable Banach space. P. Enflo and subsequently Ch. Read have provided operators on separable Banach spaces, even on $\ell_1$, which fail the invariant subspace property. Recently, in a joint work with R. Haydon a Banach space was presented, with the "scalar-plus-compact" property which, among others, yields that every every operator on the space satisfies the invariant subspace property. The space is non-reflexive and also all the Enflo-Read examples concern operators on non-reflexive spaces. The aim of the talk is to present a reflexive space such that every operator satisfies the invariant subspace property. More precisely, in a recent work with P. Motakis, using norms saturated under constraints we construct a reflexive Banach space $\mathfrak{X}_{\text{ISP}}$ which is Hereditarily Indecomposable and for every $Y$ infinite dimensional closed subspace of it and every $T \in \mathcal{L}(Y)$, $T$ admits a non trivial invariant subspace.

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**Achim Clausing**, Universität Münster, Germany.

*Pòlya operators and convexity*

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George Pòlya in a classical paper from 1931 considered the question how to choose sets of two-point boundary conditions of the form $f \to f^{(i)}(x)$, $x \in \{a, b\}$, in such a way that the differential operator $L = D^n : f \to f^{(n)}$ on $C^n[a, b]$ is invertible with regard to these conditions. We consider the analogous question for the class of disconjugate differential operators and show that these operators together with the resulting Pòlya-type boundary conditions define function cones having a remarkably rich structure. In particular, the corresponding Green's kernel is totally positive.

**Apostolos Giannopoulos**, University of Athens, Greece.

*$M$ and $M^*$-estimates for centrally symmetric convex bodies*

Given a centrally symmetric convex body $K$ in $\mathbb{R}^n$ we define

$$M(K) = \int_{S^{n-1}} \|x\| \, d\sigma(x) \text{ and } M^*(K) = \int_{S^{n-1}} h_K(x) \, d\sigma(x),$$

where $\| \cdot \|$ is the norm induced to $\mathbb{R}^n$ from $K$, $h_K$ is the support function of $K$ and $\sigma$ is the rotationally invariant probability measure on the unit Euclidean sphere $S^{n-1}$. The quantities $M$ and $M^*$ play an important role in the local theory of normed spaces. We discuss upper bounds for $M(K)$ and $M^*(K)$, mainly when $K$ is in the isotropic position.

**Peter Gruber**, TU Wien, Austria.

*Normal bundle cones of convex bodies*

The normal bundle of an ($o$-symmetric, smooth and strictly) convex body $C$ in $\mathbb{E}^d$ gives rise to a closed convex cone $\mathcal{N}_C$ in $\mathbb{E}^{d^2}$. This talk deals with properties of the normal bundle cone $\mathcal{N}_C$ and their relations to properties of the convex body $C$:

The family of normal bundle cones is a “small” subset of the space of all closed convex cones in $\mathbb{E}^{d^2}$. To determine whether a cone is a normal bundle cone, a simple criterion is given. The dimension of a normal bundle cone $\mathcal{N}_C$ is between $\frac{1}{2}d(d+1)$ and $d^2$. The lower bound is attained precisely for ellipsoids. With dimension of $\mathcal{N}_C$ increasing, the ellipsoid character of the convex body $C$ decreases. For $d = 2, 3$ a complete description of the situation is given. Next, symmetry properties are studied. The cone $\mathcal{N}_C$ coincides with its polar, its polar transpose, or its transpose, if and only if $C$ is a ball or an ellipsoid. A relation between symmetry properties of $\mathcal{N}_C$ and orthogonality in normed spaces is stated. There is a bijection between the family of certain faces of $\mathcal{N}_C$ and the family of planar shadow boundaries of $C$ with respect to parallel illumination. Finally, a relation of $\mathcal{N}_C$ and the kissing number of lattice packings of $C$ of locally ultra-maximum density is indicated.

**Martin Henk**, Universität Magdeburg, Germany.

*Successive minima inequalities*

We survey on some classical and new inequalities involving Minkowski’s successive minima $\lambda_i(K)$ of a $o$-symmetric convex body $K \subset \mathbb{R}^n$, where $\lambda_i(K)$ is the smallest positive number $\lambda$ such that $\lambda K$ contains at least $i$ linearly independent lattice points of $\mathbb{Z}^n$. Minkowski proved bounds for the volume of $K$ in terms of the successive minima, and here we want to discuss possible extensions/generalizations of these inequalities when the volume is replaced by the lattice point enumerator or when the successive minima are subject to certain restrictions.

**Menelaos Karavelas**, University of Crete, Greece.

*Exact bounds on the number of faces of the Minkowski sum of convex polytopes*

In this talk we will consider the following problem: given $r$ convex $d$-polytopes $P_1, \ldots, P_r$ in $\mathbb{R}^d$, what is the maximum number of $k$-faces of their Minkowski sum, for all $-1 \leq k \leq d-1$?

We will concentrate on the case $r = 2$ (two $d$-polytopes), and talk about exact tight worst-case bounds for all $k$'s. We will hint on the ideas behind our on-going work for the case $r = 3$ (three $d$-polytopes) and $r \geq 4$. Time permitting, we will also discuss a lower bound construction that produces a tight asymptotic expression for the complexity of the Minkowski sum when $2 \leq r \leq d-1$.

Past and on-going work jointly done with Eleni Tzanaki and Christos Konaxis.

**Alexander Koldobsky**, University of Missouri, USA.

*Stability and separation in volume comparison problems*

We prove stability and separation in several volume comparison problems, and show how these results imply hyperplane inequalities for special classes of bodies.

**Mihalis Kolountzakis**, University of Crete, Greece.

*In which domains can one do Fourier Analysis?*

We all know how to do Fourier Analysis (FA) on an interval, say $[0, 1]$, with the complete orthonormal set of functions $e^{2\pi i n x}$, $n \in \mathbb{Z}$. We also know how to use the orthogonal exponentials $e^{2\pi i (m,n)\cdot(x,y)}$, $(m, n) \in \mathbb{Z}^2$, to do FA on the square $[0, 1]^2$. Some of us may even know how to do FA on the set $[0, 1/2] \cup [1, 3/2]$ using the exponentials $e^{2\pi i (2n) x}$ and $e^{2\pi i (2n - \frac{1}{2}) x}$, $n \in \mathbb{Z}$. Yet this is not possible with the set $[0, 1/2] \cup [3/4, 5/4]$ or with the unit disk in two dimensions. So which domains are *spectral*, meaning that we can find a set of frequencies such that the corresponding exponentials are orthogonal and complete? We report on this problem, mostly its history and some recent results.

**Alexander Litvak**, University of Alberta, Canada.

*Vertex index of symmetric convex bodies*

We discuss several results on the vertex index of a given $d$-dimensional centrally symmetric convex body, which, in a sense, measures how well the body can be inscribed into a convex polytope with small number of vertices. This index is closely connected to the illumination parameter of a body, introduced earlier by Karoly Bezdek, and, thus, related to the famous conjecture in Convex Geometry about covering of a $d$-dimensional body by $2^d$ smaller positively homothetic copies. We provide estimates of this index and relate the lower bound with the outer volume ratio. We also discuss sharpness of the bounds, providing examples. The talk is based on joint works with K. Bezdek and E. D. Gluskin.

**Michel Marias**, University of Thessaloniki, Greece.

*Spectral multipliers on locally symmetric spaces*

We present the results of a joint work with N. Lohoué on spectral multipliers on locally symmetric spaces.

Denote by $G$ a connected, noncompact, real semisimple Lie group with finite centre and by $K$ a maximal compact subgroup thereof. Denote by $X$ the Riemannian symmetric space $G/K$. Let $\Gamma$ be a discrete, torsion free subgroup of $G$ and denote by $M$ the locally symmetric space $\Gamma\backslash X = \Gamma\backslash G/K$.

Given a $K$-bi-invariant kernel $\kappa$ on $G$, consider the operator

$$S_\kappa u(x) = \int_G u(g)\, \kappa\left(g^{-1}x\right) dg, \quad x \in G,\ u \in C_0^\infty(M).$$

We give a criterion of Mihlin-Hörmander type on the spherical Fourier transform of $\kappa$ that ensures the boundedness of $S_\kappa$ on $L^p(M)$.

This result reduces to a weak form of the celebrated result of J.-Ph. Anker (1990) in the case where $\Gamma$ is trivial.

**Lefteris Markessinis**, University of Athens, Greece.

*Classical positions of centrally symmetric convex bodies*

We discuss estimates of various geometric parameters of a centrally symmetric convex body which is in one of its classical positions: minimal surface area position, minimal mean width position, John position, Löwner position and the isotropic position. Using the isotropic characterizations of all these positions we provide upper bounds for the "trace distance" of any two of them.

**Mathieu Meyer**, Université Paris-Est Marne-la-Vallée, France.

*An application of shadow systems to Mahler's Conjecture*

We elaborate on the use of shadow systems to prove a particular case of the conjectured lower bound of the volume product of a convex body. In particular, we show that if a 3 dimensional convex body K is the convex hull of 2 convex bodies of dimension 2, then K satisfies Mahler conjecture both in the general case and in the centrally symmetric case (in collaboration with M. Fradelizi and A Zvavitch).

**Emanuel Milman**, Technion-Israel Institute of Technology, Israel.

*A generalization of Caffarelli's Contraction Theorem via (reverse) Heat-Flow*

A theorem of L. Caffarelli implies the existence of a map $T$, pushing forward a source Gaussian measure to a target measure which is more log-concave than the source one, which contracts Euclidean distance (in fact, Caffarelli showed that the optimal-transport Brenier map is a contraction in this case). This theorem has found numerous applications pertaining to correlation inequalities, isoperimetry, spectral-gap estimation, properties of the Gaussian measure and more. We generalize this result to more general source and target measures, using a condition on the third derivative of the potential. Contrary to the non-constructive optimal-transport map, our map $T$ is constructed as a flow along an advection field associated to an appropriate heat-diffusion process. The contraction property is then reduced to showing that log-concavity is preserved along the corresponding diffusion semi-group, by using a maximum principle for parabolic PDE. In particular, Caffarelli's original result immediately follows by using the Ornstein-Uhlenbeck process and the Prékopa-Leindler Theorem. We thus avoid using Caffarelli's regularity theory for the Monge-Ampère equation, lending our approach to further generalizations. As applications, we obtain new correlation and isoperimetric inequalities.

**Vitali Milman**, Tel-Aviv University, Israel.

*The reasons behind some classical constructions in analysis*

**Alain Pajor**, Université Paris-Est Marne-la-Vallée, France.

*Weak and strong moments estimates for convex measures*

We discuss the comparison of weak and strong moments of random vectors in an Euclidean space, whose distribution is a convex measure.

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**Rolf Schneider**, Universität Freiburg, Germany.

*Canonical diametric completions in normed spaces*

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A bounded set $M$ in a metric space $X$ is called diametrically complete if it cannot be enlarged without increasing its diameter. (In a Euclidean space, the diametrically complete sets are precisely the convex bodies of constant width.) Every nonempty bounded set is contained in a diametrically complete set of the same diameter, called a completion of the set. Generally, a set has many different completions. The usual constructions of completions make use of infinite iteration procedures, with many free choices, and thus can lead to many different completions of the same set. In contrast, we are interested in "canonical" completions, leading to a unique completion, for which it can be shown, in addition, that it depends continuously on the given set. We study such canonical completions in normed spaces (mostly, but not exclusively, of finite dimension). A further topic are general properties of the space of all diametrically complete sets, and the question when it coincides with the set of bodies of constant width. (Joint work with José Pedro Moreno)

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**Antonis Tsolomitis**, University of the Aegean, Greece.

*Random polytopes in convex bodies*

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We will present results that describe the "shape" of a random polytope with vertices in a convex body (by comparing it with deterministically constructed bodies), as well as volumetric characteristics, such as volume, surface area, mean width, etc.

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**Aljoša Volčič**, University of Calabria, Italy.

*Iterations of Steiner symmetrizations*

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To fix Steiner's proof of the isoperimetric problem, W. Gross constructed, given a convex body K, a sequence of directions $\{u_n\}$ in order to minimize the perimeter and make the corresponding Steiner symmetrizations of K converge, in the Hausdorff distance, to the ball $K^*$ centered at the origin and having the same volume as K.

Peter Mani was the first to understand (in 1986) that the convergence of the successive Steiner symmetrizations of a convex body $K$ to $K^*$ holds *almost surely.*

He conjectured that this happens also if we consider successive Steiner symmetrizations of a *compact set.* The conjecture was confirmed in 2006 by van Schaftingen. More recently we gave another proof of the conjecture. It is based on the following result.
**Theorem** *Let $F$ be a measurable set having finite measure and $P$ a symmetric probability on $S^{N-1}$ which is strictly positive on each open set. If the directions are independent and random with respect to $P$, then almost surely the iterated Steiner symmetrizations of $F$ converge in symmetric difference distance to the ball $F^*$.*

Recently Bianchi, Klain, Lutwak, Yang and Zhang (in [BKLYZ]) and, independently, Gronchi, proved that density alone of the sequence of direc- tions fung in which the Steiner symmetrizations are taken is not sufficient for the convergence. In contrast Klain proved that a sequence of Steiner sym- metrizations that uses only finitely many distinct directions always converges (but not necessarily to $K^*$). In a recent paper we (Bianchi, Burchard, Gronchi and V.), addressed several questions that were raised in these papers. Our main result says that many of the known non-converging examples do converge if the Steiner symmetrizations are followed by suitable rotations. Such behavior of the sequence is called *convergence in shape*. This confirms a conjecture posed in [BKLYZ]. The limit is, in general, not an ellipsoid (or a convex set) unless the sequence starts from an ellipsoid (or a convex set, respectively).

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**Beatrice-Helen Vritsiou**, University of Athens, Greece.

*A proof of the Bourgain-Milman inequality via the isotropic position*

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The classical Blaschke-Santaló inequality states the following: let $K$ be a convex body in $\mathbb{R}^n$, whose barycentre is at the origin, and write $K^\circ$ for its polar body; then the volume product $|K||K^\circ|$ is less than or equal to the square of the volume of the unit Euclidean ball $B_2^n$. The reverse Santaló or Bourgain-Milman inequality states then that, if we take $n$-th roots, the two volume products are approximately equal:

$$(*) \qquad |B_2^n|^{2/n} \geq (|K||K^\circ|)^{1/n} \geq c|B_2^n|^{2/n}$$

where $c > 0$ is an absolute constant (independent of the body $K$ or the dimension $n$).

Given that the volume product $|K||K^\circ|$ is an invariant of the class of linear images of $K$, we may try to prove the right-hand side of $(*)$ assuming that $K$ is isotropic and taking advantage of properties of the isotropic position, which has been studied extensively. Recall that a convex body $K$ in $\mathbb{R}^n$ is called isotropic if it has volume 1, its barycentre is at the origin, and if there exists a constant $L_K$ such that for every $y \in \mathbb{R}^n$,

$$\int_K \langle x, y\rangle^2 dx = (L_K \|y\|_2)^2.$$

The aim of this talk is to present, as self-containedly as possible, one such proof of the Bourgain-Milman inequality which exploits only basic tools from the Asymptotic Theory of Convex Bodies and Log-concave Measures.

Joint work with A. Giannopoulos and G. Paouris.

**Participants**

Gergely Ambrus, Renyi Institute, Hungary.
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Spiros Argyros, National Technical University of Athens, Greece.
Georgia Athanasaki, University of Crete, Greece.
Silouanos Brazitikos, University of Athens, Greece.
Ingrid Carbone, University of Calabria, Italy.
Athanasios Chatzikaleas, University of Crete, Greece.
Achim Clausing, Universitat Münster, Germany.
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Emmanuel Katsoprinakis, University of Crete, Greece.
Alexander Koldobsky, University of Missouri, USA.
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Shlomo Reisner, University of Haifa, Israel.
Konstantinos Rousohatzakis, University of Crete, Greece.
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**Secretary Dept of Math**: (+30) 2810-393800

## SOME RESTAURANTS (*=more expensive)

*Near the Old Port*

**Parasies**, Square of Historical Museum.

**Ippokambos Ouzeri**, Sofokli Venizelou 3.

**O Vrakas**, Marineli 1.

**I Avli tou Defkaliona**, Prevelaki 10.

**Veneto**, 9 Epimenidou St.

**Brillant***, 15 Epimenidou St.

**Loukoulos***, Korai St.

*In the City*

**Erganos**, Georgiadi 5, Oasi.

**Kiriakos**, Leoforos Dimokratias 53.

**Kipos ton Gefseon**, 8 Chrisostomou Av.

**Merastri**, Chrysostomou 17.

**Empolo**, M. Miliara 7.