

A Bendixson-Dulac type criterion for non-periodic minimal sets and related problems

Abstract

I shall present a non-existence theorem for non-periodic compact minimal sets of C^1 vector fields on orientable smooth surfaces, which is analogous to the Bendixson-Dulac criterion on the non-existence of periodic solutions for planar ODE's. As corollary we get that the divergence of the C^1 vector field with respect to any area 2-form always has a vanishing point on a given non-periodic compact minimal set. This leads to the question whether the divergence with respect to *some* area 2-form vanishes identically on the minimal set. The latter is equivalent to the problem whether the divergence with respect to any area 2-form is a Whitney regular continuous coboundary on the minimal set. In the case of the 2-torus the problem reduces to the study of a discrete cohomological equation on a Cantor set and can be interpreted as a question on the existence of an absolutely continuous conformal probability measure on the unique Cantor minimal set of a Denjoy C^1 diffeomorphism of the circle.

References

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- [2] K. Athanassopoulos, On the existence of absolutely continuous conformal measures for uniquely ergodic minimal Cantor homeomorphisms, preprint.