Turn in your solutions by 7/4/2020. See directions in the class webpage.

1. In the lecture we showed that if $f \in C(\mathbb{T})$ has non-zero Fourier coefficients only on the powers of 3 then $S_N f$ converges to f uniformly on $\mathbb{T}$.

Prove the same if the Fourier coefficients of f are non-zero only at the locations $\pm n_1, \pm n_2, \pm n_3, \ldots$, with $1 \le n_1 < n_2 < n_3 < \cdots$, where $\frac{n_{k+1}}{n_k} \ge \rho > 1$, for $k \ge 1$.

2. Define the function $f : \mathbb{R} \to \mathbb{R}$ to be 0 on the irrationals and at 0 and to be equal to $1/n$ on every rational of the form m/n with $(m, n) = 1$. Show that f is continuous exactly on the irrationals and at 0.

3. The function $f : \mathbb{R} \to \mathbb{R}$ is increasing. Show that there exists a countable set $E \subseteq \mathbb{R}$, possibly empty, such that f is continuous on $\mathbb{R} \setminus E$.