Turn in your solutions by 20/5/2020. See directions in the class webpage.

1. If $E \subseteq [0, 2\pi]$ and $\xi_n \in \mathbb{R}$ is any sequence show that

$$\int_E \cos^2(nx + \xi_n)\, dx \to \frac{1}{2}|E|.$$

Use the Riemann-Lebesgue Lemma .

2. If $0 < \alpha < \beta < 1$ construct a function f which is Lip-α but not Lip-β.

3. If a function $f \in C(\mathbb{T})$ is Lipschitz-α for some $\alpha > 1$ show that the function is necessarily constant.

If $x \neq y$ show that $g(x) = g(y)$ writing

$$|g(x) - g(y)| \leq |g(x) - g(x + \delta)| + |g(x + \delta) - g(x + 2\delta)| + \cdots + |g(x + (n - 1)\delta) - g(y)|,$$

where $\delta = (y - x)/n$.