Turn in your solutions in class on Thursday 5/3/2020. **Turn in also the previous set that you forgot to turn in this Thursday.** Write briefly without ommitting the essentials.

1. If $f \in L^1(\mathbb{T})$, $g \in C(\mathbb{T})$ show that $f * g \in C(\mathbb{T})$.

Write $f * g(x_0) - f * g(x_0 + h)$ as an integral and use the Dominated Convergence Theorem in order to show that it goes to 0 for $h \to 0$.

2. If $f \in L^1(\mathbb{T})$, $g \in C^1(\mathbb{T})$ show that $f * g \in C^1(\mathbb{T})$ and that

$$(f * g)' = f * g'.$$

Express the difference $f*g'(x_0) - \frac{1}{h}(f*g(x_0+h) - f*g(x_0))$ as an integral and use the Dominated Convergence Theorem to show that it goes to 0 as $h \to 0$.

3. Find a closed formula for the Dirichlet kernel of order N:

$$D_N(x) = \sum_{k=-N}^{N} e^{ikx}.$$

Use the formula for the finite geometric series $1 + a + a^2 + a^3 + \cdots + a^{n-1} = \frac{1-a^n}{1-a}$, for $a \in \mathbb{C} \setminus \{1\}$.