University of Crete – Department of Mathematics and Applied Mathematics
Problem Set No 9

Turn in your solutions in class on Thursday 12/3/2020. Write briefly without ommitting the essentials.

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1. Show that if $K_N(x) = \sum_{k=-N}^{N}\left(1 - \frac{|k|}{N+1}\right) e^{ikx}$ then

$$K_N(x) = \frac{1}{N+1} \frac{\sin^2 \frac{(N+1)x}{2}}{\sin^2 \frac{x}{2}}.$$

2. If $f \in L^1(\mathbb{T})$ and $N \in \mathbb{N}$ find the Fourier coefficients of the function $f(Nx)$ via those of $f(x)$.

3. If $f \in L^1(\mathbb{T})$ and $g \in L^\infty(\mathbb{T})$ show that

$$\lim_{n\to\infty} \frac{1}{2\pi} \int_0^{2\pi} f(t) g(nt)\, dt = \widehat{f}(0)\widehat{g}(0).$$

Show it first when f is a trigonometric polynomial. Then use the density of trigonometric polynomials in $L^1(\mathbb{T})$. Problem 2 will be useful.

Harmonic Analysis - Spring 2019-20 - M. Kolountzakis