

Serial Number: **500**, Answers: 1: 2: 3: 4: 5: 6: 7: 8: 9:

Name:

UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $10!$  B:  $3^{11}$  C:  $11!$  D:  $9!$

**Question 2:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ . B: side  $B$  has more vertices than side  $A$ . C: side  $A$  has more vertices than side  $B$ . D: there is always a perfect matching of the vertices of side  $A$ .

**Question 3:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $n^m$  B:  $m^n$  C:  $n(n-1) \cdots (n-m+1)$  D:  $m \cdot n$

**Question 4:** The binomial coefficient  $\binom{n}{k}$  equals

A: 0 if  $k = 0$ . B:  $\binom{n}{n-k}$ .

**Question 5:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $20^3$  B:  $\frac{20!}{3!}$  C:  $20 \cdot 19 \cdot 18$  D:  $3^{20}$

**Question 6:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 7:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $2(m+n)$  B:  $m+n$  C:  $m(n-1) + n(m-1)$  D:  $m \cdot n$

**Question 8:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $2^{10}$  B:  $10 \times 10$  C:  $10!$  D:  $11!$

**Question 9:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: each vertex of side  $B$  is connected to some vertex in side  $A$ . B: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . C: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . D: each vertex of side  $A$  is connected with all vertices of side  $B$ .

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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Name:

UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $3^{10}$  B:  $10 \cdot 9 \cdot 8$  C:  $10^3$  D: 30

**Question 2:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $2^{10}$  B:  $10 \times 10$  C:  $10!$  D:  $11!$

**Question 3:** In a simple graph with 100 vertices

A: not all vertex degrees can be odd. B: the minimum vertex degree is  $\geq 1$ . C: it is possible that all vertices have different degrees. D: the maximum vertex degree is  $\leq 99$ .

**Question 4:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{n(n-1)\cdots(n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$  B:  $\frac{k(k-1)\cdots(k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$

**Question 5:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to the degree of vertex  $i$  B: equal to 1 exactly when there is a path that connect  $i$  to  $j$ . C: equal to 1 exactly when  $i$  is not connected to  $j$  D: equal to 0 exactly when  $i$  is not connected to  $j$

**Question 6:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $n^m$  B:  $m \cdot n$  C:  $n(n-1) \cdots (n-m+1)$  D:  $m^n$

**Question 7:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

**Question 8:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!}$  B:  $6!$  C:  $\frac{10!}{6!4!}$  D:  $10^4$

**Question 9:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it cannot have more than  $n + 1$  edges. B: it must have at least  $n - 1$  edges. C: it must have at least  $n$  edges. D: it cannot contain cycles.

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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Name:

UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $10!$  B:  $11!$  C:  $9!$  D:  $3^{11}$

**Question 2:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

**Question 3:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1)\cdots(k-n+1)}{n\cdot(n-1)\cdots 2\cdot 1}$  B:  $\frac{n(n-1)\cdots(n-k+1)}{k\cdot(k-1)\cdots 2\cdot 1}$

**Question 4:** If  $G$  is a simple graph then

A: it has at most two vertices with odd degree. B: it has at least two vertices with odd degree. C: the number of its vertices with odd degree is not odd. D: the number of its vertices with even degree is even.

**Question 5:** In a simple graph with 100 vertices

A: not all vertex degrees can be odd. B: the maximum vertex degree is  $\leq 99$ . C: the minimum vertex degree is  $\geq 1$ . D: it is possible that all vertices have different degrees.

**Question 6:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $10 \cdot 9 \cdot 8$  B:  $10^3$  C:  $3^{10}$  D: 30

**Question 7:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $2^n$  B:  $2^n + 2^n$  C:  $3^n$  D:  $\binom{n}{n/2}$

**Question 8:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!4!}$  B:  $6!$  C:  $\frac{10!}{6!}$  D:  $10^4$

**Question 9:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: there is always a perfect matching of the vertices of side  $A$ . B: side  $B$  has more vertices than side  $A$ . C: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ . D: side  $A$  has more vertices than side  $B$ .

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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Name:

UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $11!$  B:  $10!$  C:  $2^{10}$  D:  $10 \times 10$

**Question 2:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $\binom{n}{n/2}$  B:  $2^n$  C:  $3^n$  D:  $2^n + 2^n$

**Question 3:** How many different quadruples can one form from the objects  $1, 1, 2, 3, 4, 5, 6, 7, 8, 9$ . Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

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A: 0 if  $k = 0$ . B:  $\binom{n}{n-k}$ .

**Question 5:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it cannot have more than  $n + 1$  edges. B: it cannot contain cycles. C: it must have at least  $n - 1$  edges. D: it must have at least  $n$  edges.

**Question 6:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $11!$  B:  $3^{11}$  C:  $10!$  D:  $9!$

**Question 7:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m \cdot n$  B:  $2(m + n)$  C:  $m(n - 1) + n(m - 1)$  D:  $m + n$

**Question 8:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $n(n - 1) \cdots (n - m + 1)$  B:  $n^m$  C:  $m \cdot n$  D:  $m^n$

**Question 9:** In a simple graph with 100 vertices

A: the minimum vertex degree is  $\geq 1$ . B: not all vertex degrees can be odd. C: it is possible that all vertices have different degrees. D: the maximum vertex degree is  $\leq 99$ .

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** If  $G$  is a connected simple graph with  $n$  vertices then

$A$ : it must have at least  $n - 1$  edges.  $B$ : it cannot contain cycles.  $C$ : it must have at least  $n$  edges.

$D$ : it cannot have more than  $n + 1$  edges.

**Question 2:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

$A$ :  $9!$   $B$ :  $3^{11}$   $C$ :  $10!$   $D$ :  $11!$

**Question 3:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

$A$ :  $3^{20}$   $B$ :  $\frac{20!}{3!}$   $C$ :  $20^3$   $D$ :  $20 \cdot 19 \cdot 18$

**Question 4:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

$A$ :  $\frac{k(k-1)\cdots(k-n+1)}{n(n-1)\cdots 2 \cdot 1}$   $B$ :  $\frac{n(n-1)\cdots(n-k+1)}{k(k-1)\cdots 2 \cdot 1}$

**Question 5:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

$A$ :  $m^n$   $B$ :  $m \cdot n$   $C$ :  $n(n-1)\cdots(n-m+1)$   $D$ :  $n^m$

**Question 6:** In a simple graph with 100 vertices

$A$ : it is possible that all vertices have different degrees.  $B$ : not all vertex degrees can be odd.  $C$ : the maximum vertex degree is  $\leq 99$ .  $D$ : the minimum vertex degree is  $\geq 1$ .

**Question 7:** If  $G$  is a simple graph then

$A$ : it has at most two vertices with odd degree.  $B$ : the number of its vertices with odd degree is not odd.

$C$ : the number of its vertices with even degree is even.  $D$ : it has at least two vertices with odd degree.

**Question 8:** The binomial coefficient  $\binom{n}{k}$  equals

$A$ : 0 if  $k = 0$ .  $B$ :  $\binom{n}{n-k}$ .

**Question 9:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

$A$ :  $2^{10}$   $B$ :  $11!$   $C$ :  $10!$   $D$ :  $10 \times 10$

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Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m \cdot n$  B:  $2(m+n)$  C:  $m+n$  D:  $m(n-1) + n(m-1)$

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A:  $3^{11}$  B:  $10!$  C:  $11!$  D:  $9!$

**Question 4:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $10^4$  B:  $6!$  C:  $\frac{10!}{6!4!}$  D:  $\frac{10!}{6!}$

**Question 5:** In a simple graph with 100 vertices

A: the minimum vertex degree is  $\geq 1$ . B: it is possible that all vertices have different degrees. C: the maximum vertex degree is  $\leq 99$ . D: not all vertex degrees can be odd.

**Question 6:** If  $G$  is a simple graph then

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A:  $\frac{k(k-1)\dots(k-n+1)}{n \cdot (n-1) \dots 2 \cdot 1}$  B:  $\frac{n(n-1)\dots(n-k+1)}{k \cdot (k-1) \dots 2 \cdot 1}$

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A:  $2^n$  B:  $2^n + 2^n$  C:  $3^n$  D:  $\binom{n}{n/2}$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $20^3$  B:  $20 \cdot 19 \cdot 18$  C:  $3^{20}$  D:  $\frac{20!}{3!}$

**Question 2:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: each vertex of side  $B$  is connected to some vertex in side  $A$ . B: each vertex of side  $A$  is connected with all vertices of side  $B$ . C: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . D: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .

**Question 3:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

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**Question 5:** The binomial coefficient  $\binom{n}{k}$  equals

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**Question 6:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m(n-1) + n(m-1)$  B:  $2(m+n)$  C:  $m+n$  D:  $m \cdot n$

**Question 7:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!}$  B:  $10^4$  C:  $6!$  D:  $\frac{10!}{6!4!}$

**Question 8:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $10 \cdot 9 \cdot 8$  B:  $3^{10}$  C: 30 D:  $10^3$

**Question 9:** In a simple graph with 100 vertices

A: the maximum vertex degree is  $\leq 99$ . B: it is possible that all vertices have different degrees. C: the minimum vertex degree is  $\geq 1$ . D: not all vertex degrees can be odd.

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Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

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A:  $m \cdot n$  B:  $m(n-1) + n(m-1)$  C:  $m+n$  D:  $2(m+n)$

**Question 3:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k=0$ .

**Question 4:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to 1 exactly when  $i$  is not connected to  $j$  B: equal to the degree of vertex  $i$  C: equal to 0 exactly when  $i$  is not connected to  $j$  D: equal to 1 exactly when there is a path that connect  $i$  to  $j$ .

**Question 5:** If  $G$  is a simple graph then

A: the number of its vertices with even degree is even. B: the number of its vertices with odd degree is not odd. C: it has at most two vertices with odd degree. D: it has at least two vertices with odd degree.

**Question 6:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $m^n$  B:  $n(n-1) \cdots (n-m+1)$  C:  $m \cdot n$  D:  $n^m$

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A:  $11!$  B:  $9!$  C:  $3^{11}$  D:  $10!$

**Question 8:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

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A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

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Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

$$A: \frac{k(k-1)\cdots(k-n+1)}{n\cdot(n-1)\cdots 2\cdot 1} \quad B: \frac{n(n-1)\cdots(n-k+1)}{k\cdot(k-1)\cdots 2\cdot 1}$$

**Question 2:** In a simple graph with 100 vertices

$A$ : not all vertex degrees can be odd.  $B$ : the minimum vertex degree is  $\geq 1$ .  $C$ : the maximum vertex degree is  $\leq 99$ .  $D$ : it is possible that all vertices have different degrees.

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$$A: n(n-1)\cdots(n-m+1) \quad B: m^n \quad C: n^m \quad D: m \cdot n$$

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$$A: m+n \quad B: 2(m+n) \quad C: m \cdot n \quad D: m(n-1) + n(m-1)$$

**Question 5:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

$$A: 3^{11} \quad B: 11! \quad C: 9! \quad D: 10!$$

**Question 6:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

$A$ : each vertex of side  $A$  is connected with all vertices of side  $B$ .  $B$ : the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .  $C$ : each vertex of side  $B$  is connected to some vertex in side  $A$ .  $D$ : the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .

**Question 7:** How many different quadruples can one form from the objects  $1, 1, 2, 3, 4, 5, 6, 7, 8, 9$ . Two quadruples differing only in order are not considered different.

$$A: \binom{8}{4} + \binom{8}{3} + \binom{8}{2} \quad B: 10 \cdot 9 \cdot 8 \cdot 7$$

**Question 8:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

$$A: 2^{10} \quad B: 10! \quad C: 10 \times 10 \quad D: 11!$$

**Question 9:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

$$A: 3^{20} \quad B: 20 \cdot 19 \cdot 18 \quad C: 20^3 \quad D: \frac{20!}{3!}$$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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Name:

UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** If  $G$  is a simple graph then

$A$ : it has at most two vertices with odd degree.  $B$ : the number of its vertices with even degree is even.  
 $C$ : it has at least two vertices with odd degree.  $D$ : the number of its vertices with odd degree is not odd.

**Question 2:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

$A$ :  $\frac{20!}{3!}$   $B$ :  $20^3$   $C$ :  $3^{20}$   $D$ :  $20 \cdot 19 \cdot 18$

**Question 3:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

$A$ :  $\frac{10!}{6!4!}$   $B$ :  $6!$   $C$ :  $10^4$   $D$ :  $\frac{10!}{6!}$

**Question 4:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

$A$ :  $n(n-1) \cdots (n-m+1)$   $B$ :  $n^m$   $C$ :  $m \cdot n$   $D$ :  $m^n$

**Question 5:** The binomial coefficient  $\binom{n}{k}$  equals

$A$ : 0 if  $k = 0$ .  $B$ :  $\binom{n}{n-k}$ .

**Question 6:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

$A$ :  $10 \cdot 9 \cdot 8 \cdot 7$   $B$ :  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 7:** How many circular orderings of the numbers 0, 1, ..., 10 are there? (Two circular orderings which differ only by a rotation are not considered different.)

$A$ :  $3^{11}$   $B$ :  $11!$   $C$ :  $10!$   $D$ :  $9!$

**Question 8:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

$A$ : For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ .  $B$ : side  $B$  has more vertices than side  $A$ .  $C$ : there is always a perfect matching of the vertices of side  $A$ .  $D$ : side  $A$  has more vertices than side  $B$ .

**Question 9:** If  $G$  is a connected simple graph with  $n$  vertices then

$A$ : it must have at least  $n$  edges.  $B$ : it cannot contain cycles.  $C$ : it must have at least  $n - 1$  edges.  
 $D$ : it cannot have more than  $n + 1$  edges.

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** If  $G$  is a connected simple graph with  $n$  vertices then

$A$ : it cannot have more than  $n + 1$  edges.  $B$ : it must have at least  $n - 1$  edges.  $C$ : it cannot contain cycles.  $D$ : it must have at least  $n$  edges.

**Question 2:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

$A$ : equal to the degree of vertex  $i$   $B$ : equal to 1 exactly when  $i$  is not connected to  $j$   $C$ : equal to 0 exactly when  $i$  is not connected to  $j$   $D$ : equal to 1 exactly when there is a path that connect  $i$  to  $j$ .

**Question 3:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

$A$ :  $20^3$   $B$ :  $3^{20}$   $C$ :  $20 \cdot 19 \cdot 18$   $D$ :  $\frac{20!}{3!}$

**Question 4:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

$A$ : side  $B$  has more vertices than side  $A$ .  $B$ : For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ .  $C$ : side  $A$  has more vertices than side  $B$ .  $D$ : there is always a perfect matching of the vertices of side  $A$ .

**Question 5:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

$A$ :  $6!$   $B$ :  $10^4$   $C$ :  $\frac{10!}{6!}$   $D$ :  $\frac{10!}{6!4!}$

**Question 6:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

$A$ :  $10!$   $B$ :  $11!$   $C$ :  $2^{10}$   $D$ :  $10 \times 10$

**Question 7:** The binomial coefficient  $\binom{n}{k}$  equals

$A$ : 0 if  $k = 0$ .  $B$ :  $\binom{n}{n-k}$ .

**Question 8:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

$A$ :  $10^3$   $B$ :  $3^{10}$   $C$ : 30  $D$ :  $10 \cdot 9 \cdot 8$

**Question 9:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

$A$ :  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$   $B$ :  $10 \cdot 9 \cdot 8 \cdot 7$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $n(n-1) \cdots (n-m+1)$  B:  $m^n$  C:  $m \cdot n$  D:  $n^m$

**Question 2:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1) \cdots (k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$  B:  $\frac{n(n-1) \cdots (n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$

**Question 3:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: side  $A$  has more vertices than side  $B$ . B: there is always a perfect matching of the vertices of side  $A$ . C: side  $B$  has more vertices than side  $A$ . D: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ .

**Question 4:** If  $G$  is a simple graph then

A: it has at least two vertices with odd degree. B: the number of its vertices with odd degree is not odd. C: it has at most two vertices with odd degree. D: the number of its vertices with even degree is even.

**Question 5:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $11!$  B:  $9!$  C:  $10!$  D:  $3^{11}$

**Question 6:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!4!}$  B:  $10^4$  C:  $6!$  D:  $\frac{10!}{6!}$

**Question 7:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $2^n$  B:  $\binom{n}{n/2}$  C:  $3^n$  D:  $2^n + 2^n$

**Question 8:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . B: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . C: each vertex of side  $A$  is connected with all vertices of side  $B$ . D: each vertex of side  $B$  is connected to some vertex in side  $A$ .

**Question 9:** How many different quadruples can one form from the objects  $1, 1, 2, 3, 4, 5, 6, 7, 8, 9$ . Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $10!$  B:  $11!$  C:  $2^{10}$  D:  $10 \times 10$

**Question 2:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it cannot have more than  $n + 1$  edges. B: it must have at least  $n$  edges. C: it must have at least  $n - 1$  edges. D: it cannot contain cycles.

**Question 3:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $10^4$  B:  $\frac{10!}{6!4!}$  C:  $6!$  D:  $\frac{10!}{6!}$

**Question 4:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

**Question 5:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A: 30 B:  $10^3$  C:  $3^{10}$  D:  $10 \cdot 9 \cdot 8$

**Question 6:** In a simple graph with 100 vertices

A: the maximum vertex degree is  $\leq 99$ . B: not all vertex degrees can be odd. C: it is possible that all vertices have different degrees. D: the minimum vertex degree is  $\geq 1$ .

**Question 7:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to 1 exactly when there is a path that connect  $i$  to  $j$ . B: equal to 1 exactly when  $i$  is not connected to  $j$  C: equal to 0 exactly when  $i$  is not connected to  $j$  D: equal to the degree of vertex  $i$

**Question 8:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $20^3$  B:  $\frac{20!}{3!}$  C:  $3^{20}$  D:  $20 \cdot 19 \cdot 18$

**Question 9:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 2:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m + n$  B:  $m(n - 1) + n(m - 1)$  C:  $m \cdot n$  D:  $2(m + n)$

**Question 3:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1)\cdots(k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$  B:  $\frac{n(n-1)\cdots(n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$

**Question 4:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $m^n$  B:  $m \cdot n$  C:  $n^m$  D:  $n(n - 1) \cdots (n - m + 1)$

**Question 5:** How many circular orderings of the numbers 0, 1, ..., 10 are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $3^{11}$  B:  $10!$  C:  $11!$  D:  $9!$

**Question 6:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ . B: side  $B$  has more vertices than side  $A$ . C: there is always a perfect matching of the vertices of side  $A$ . D: side  $A$  has more vertices than side  $B$ .

**Question 7:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $3^{20}$  B:  $\frac{20!}{3!}$  C:  $20 \cdot 19 \cdot 18$  D:  $20^3$

**Question 8:** In how many ways can the numbers 0, 1, ..., 10 be put in order?

A:  $10 \times 10$  B:  $11!$  C:  $10!$  D:  $2^{10}$

**Question 9:** If  $G$  is a simple graph then

A: the number of its vertices with even degree is even. B: it has at most two vertices with odd degree. C: it has at least two vertices with odd degree. D: the number of its vertices with odd degree is not odd.

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

$A$ : For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ .  $B$ : there is always a perfect matching of the vertices of side  $A$ .  $C$ : side  $A$  has more vertices than side  $B$ .  $D$ : side  $B$  has more vertices than side  $A$ .

**Question 2:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

$A$ : each vertex of side  $A$  is connected with all vertices of side  $B$ .  $B$ : the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .  $C$ : each vertex of side  $B$  is connected to some vertex in side  $A$ .  $D$ : the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .

**Question 3:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

$A$ :  $11!$   $B$ :  $2^{10}$   $C$ :  $10 \times 10$   $D$ :  $10!$

**Question 4:** If  $G$  is a simple graph then

$A$ : the number of its vertices with even degree is even.  $B$ : it has at most two vertices with odd degree.  $C$ : it has at least two vertices with odd degree.  $D$ : the number of its vertices with odd degree is not odd.

**Question 5:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

$A$ :  $n^m$   $B$ :  $m^n$   $C$ :  $m \cdot n$   $D$ :  $n(n-1) \cdots (n-m+1)$

**Question 6:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

$A$ :  $9!$   $B$ :  $11!$   $C$ :  $10!$   $D$ :  $3^{11}$

**Question 7:** How many different quadruples can one form from the objects  $1, 1, 2, 3, 4, 5, 6, 7, 8, 9$ . Two quadruples differing only in order are not considered different.

$A$ :  $10 \cdot 9 \cdot 8 \cdot 7$   $B$ :  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 8:** The binomial coefficient  $\binom{n}{k}$  equals

$A$ :  $\binom{n}{n-k}$ .  $B$ : 0 if  $k = 0$ .

**Question 9:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

$A$ :  $\binom{n}{n/2}$   $B$ :  $3^n$   $C$ :  $2^n$   $D$ :  $2^n + 2^n$

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Instructor: Mihalis Kolountzakis

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** The binomial coefficient  $\binom{n}{k}$  equals

A: 0 if  $k = 0$ . B:  $\binom{n}{n-k}$ .

**Question 2:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $10^3$  B: 30 C:  $10 \cdot 9 \cdot 8$  D:  $3^{10}$

**Question 3:** If  $G$  is a simple graph then

A: it has at least two vertices with odd degree. B: the number of its vertices with even degree is even.  
C: it has at most two vertices with odd degree. D: the number of its vertices with odd degree is not odd.

**Question 4:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!}$  B:  $6!$  C:  $10^4$  D:  $\frac{10!}{6!4!}$

**Question 5:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to 1 exactly when there is a path that connect  $i$  to  $j$ . B: equal to 1 exactly when  $i$  is not connected to  $j$  C: equal to 0 exactly when  $i$  is not connected to  $j$  D: equal to the degree of vertex  $i$

**Question 6:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $n^m$  B:  $m \cdot n$  C:  $m^n$  D:  $n(n-1) \cdots (n-m+1)$

**Question 7:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: side  $B$  has more vertices than side  $A$ . B: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ . C: side  $A$  has more vertices than side  $B$ . D: there is always a perfect matching of the vertices of side  $A$ .

**Question 8:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{n(n-1) \cdots (n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$  B:  $\frac{k(k-1) \cdots (k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$

**Question 9:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $2^{10}$  B:  $10 \times 10$  C:  $11!$  D:  $10!$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $20^3$  B:  $\frac{20!}{3!}$  C:  $3^{20}$  D:  $20 \cdot 19 \cdot 18$

**Question 2:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!4!}$  B:  $6!$  C:  $10^4$  D:  $\frac{10!}{6!}$

**Question 3:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1)\cdots(k-n+1)}{n \cdot (n-1)\cdots 2 \cdot 1}$  B:  $\frac{n(n-1)\cdots(n-k+1)}{k \cdot (k-1)\cdots 2 \cdot 1}$

**Question 4:** In a simple graph with 100 vertices

A: the minimum vertex degree is  $\geq 1$ . B: the maximum vertex degree is  $\leq 99$ . C: it is possible that all vertices have different degrees. D: not all vertex degrees can be odd.

**Question 5:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it must have at least  $n$  edges. B: it cannot contain cycles. C: it cannot have more than  $n + 1$  edges. D: it must have at least  $n - 1$  edges.

**Question 6:** If  $G$  is a simple graph then

A: the number of its vertices with even degree is even. B: it has at least two vertices with odd degree. C: the number of its vertices with odd degree is not odd. D: it has at most two vertices with odd degree.

**Question 7:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $2^{10}$  B:  $10 \times 10$  C:  $11!$  D:  $10!$

**Question 8:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 9:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $10 \cdot 9 \cdot 8$  B:  $3^{10}$  C:  $30$  D:  $10^3$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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Name:

UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** If  $G$  is a simple graph then

$A$ : it has at least two vertices with odd degree.  $B$ : the number of its vertices with even degree is even.  
 $C$ : the number of its vertices with odd degree is not odd.  $D$ : it has at most two vertices with odd degree.

**Question 2:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

$A$ :  $\frac{n(n-1)\cdots(n-k+1)}{k\cdot(k-1)\cdots 2\cdot 1}$   $B$ :  $\frac{k(k-1)\cdots(k-n+1)}{n\cdot(n-1)\cdots 2\cdot 1}$

**Question 3:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

$A$ :  $10 \times 10$   $B$ :  $11!$   $C$ :  $10!$   $D$ :  $2^{10}$

**Question 4:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

$A$ : side  $B$  has more vertices than side  $A$ .  $B$ : For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ .  $C$ : side  $A$  has more vertices than side  $B$ .  $D$ : there is always a perfect matching of the vertices of side  $A$ .

**Question 5:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

$A$ :  $m \cdot n$   $B$ :  $m + n$   $C$ :  $m(n - 1) + n(m - 1)$   $D$ :  $2(m + n)$

**Question 6:** The binomial coefficient  $\binom{n}{k}$  equals

$A$ :  $\binom{n}{n-k}$ .  $B$ : 0 if  $k = 0$ .

**Question 7:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

$A$ :  $10^4$   $B$ :  $\frac{10!}{6!4!}$   $C$ :  $6!$   $D$ :  $\frac{10!}{6!}$

**Question 8:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

$A$ :  $n(n - 1) \cdots (n - m + 1)$   $B$ :  $n^m$   $C$ :  $m \cdot n$   $D$ :  $m^n$

**Question 9:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

$A$ :  $\binom{n}{n/2}$   $B$ :  $2^n$   $C$ :  $3^n$   $D$ :  $2^n + 2^n$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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Name:

UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

$A$ : For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ .  $B$ : side  $B$  has more vertices than side  $A$ .  $C$ : there is always a perfect matching of the vertices of side  $A$ .  $D$ : side  $A$  has more vertices than side  $B$ .

**Question 2:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

$A$ :  $10 \cdot 9 \cdot 8$   $B$ : 30  $C$ :  $10^3$   $D$ :  $3^{10}$

**Question 3:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

$A$ :  $\frac{k(k-1)\cdots(k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$   $B$ :  $\frac{n(n-1)\cdots(n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$

**Question 4:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

$A$ :  $6!$   $B$ :  $\frac{10!}{6!4!}$   $C$ :  $\frac{10!}{6!}$   $D$ :  $10^4$

**Question 5:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

$A$ : each vertex of side  $B$  is connected to some vertex in side  $A$ .  $B$ : the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .  $C$ : each vertex of side  $A$  is connected with all vertices of side  $B$ .  $D$ : the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .

**Question 6:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

$A$ :  $n^m$   $B$ :  $m^n$   $C$ :  $n(n-1)\cdots(n-m+1)$   $D$ :  $m \cdot n$

**Question 7:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

$A$ :  $10 \cdot 9 \cdot 8 \cdot 7$   $B$ :  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 8:** In how many ways can the numbers 0, 1,  $\dots$ , 10 be put in order?

$A$ :  $11!$   $B$ :  $10 \times 10$   $C$ :  $10!$   $D$ :  $2^{10}$

**Question 9:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

$A$ :  $m \cdot n$   $B$ :  $m + n$   $C$ :  $m(n-1) + n(m-1)$   $D$ :  $2(m+n)$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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Name:

UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$   
 $A$ : the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .  $B$ : each vertex of side  $A$   
is connected with all vertices of side  $B$ .  $C$ : each vertex of side  $B$  is connected to some vertex in side  $A$ .  
 $D$ : the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .

**Question 2:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

$A$ :  $11!$   $B$ :  $10!$   $C$ :  $10 \times 10$   $D$ :  $2^{10}$

**Question 3:** The binomial coefficient  $\binom{n}{k}$  equals

$A$ : 0 if  $k = 0$ .  $B$ :  $\binom{n}{n-k}$ .

**Question 4:** If  $G$  is a simple graph then

$A$ : the number of its vertices with even degree is even.  $B$ : it has at least two vertices with odd degree.  
 $C$ : the number of its vertices with odd degree is not odd.  $D$ : it has at most two vertices with odd  
degree.

**Question 5:** How many different quadruples can one form from the objects  $1, 1, 2, 3, 4, 5, 6, 7, 8, 9$ . Two  
quadruples differing only in order are not considered different.

$A$ :  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$   $B$ :  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 6:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

$A$ :  $m \cdot n$   $B$ :  $n^m$   $C$ :  $m^n$   $D$ :  $n(n-1) \cdots (n-m+1)$

**Question 7:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$   
and  $B = \{b_1, \dots, b_n\}$  is

$A$ :  $2(m+n)$   $B$ :  $m \cdot n$   $C$ :  $m(n-1) + n(m-1)$   $D$ :  $m+n$

**Question 8:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons  
with a chair, secretary and member?

$A$ :  $3^{20}$   $B$ :  $\frac{20!}{3!}$   $C$ :  $20 \cdot 19 \cdot 18$   $D$ :  $20^3$

**Question 9:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings  
which differ only by a rotation are not considered different.)

$A$ :  $11!$   $B$ :  $10!$   $C$ :  $9!$   $D$ :  $3^{11}$

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serial number of your paper and your answers on a piece of paper and keep it. Wrong answers reduce your score.  
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Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $10 \cdot 9 \cdot 8$  B:  $3^{10}$  C: 30 D:  $10^3$

**Question 2:** The binomial coefficient  $\binom{n}{k}$  equals

A: 0 if  $k = 0$ . B:  $\binom{n}{n-k}$ .

**Question 3:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it cannot contain cycles. B: it must have at least  $n - 1$  edges. C: it cannot have more than  $n + 1$  edges. D: it must have at least  $n$  edges.

**Question 4:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $2(m + n)$  B:  $m(n - 1) + n(m - 1)$  C:  $m \cdot n$  D:  $m + n$

**Question 5:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $10!$  B:  $3^{11}$  C:  $9!$  D:  $11!$

**Question 6:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $2^{10}$  B:  $10 \times 10$  C:  $10!$  D:  $11!$

**Question 7:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: each vertex of side  $A$  is connected with all vertices of side  $B$ . B: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . C: each vertex of side  $B$  is connected to some vertex in side  $A$ . D: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .

**Question 8:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $m^n$  B:  $n(n - 1) \cdots (n - m + 1)$  C:  $m \cdot n$  D:  $n^m$

**Question 9:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{n(n-1)\cdots(n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$  B:  $\frac{k(k-1)\cdots(k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$   
 $A$ : the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .  $B$ : each vertex of side  $B$  is connected to some vertex in side  $A$ .  $C$ : the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .  $D$ : each vertex of side  $A$  is connected with all vertices of side  $B$ .

**Question 2:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

$A$ : side  $A$  has more vertices than side  $B$ .  $B$ : there is always a perfect matching of the vertices of side  $A$ .  $C$ : side  $B$  has more vertices than side  $A$ .  $D$ : For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ .

**Question 3:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

$A$ :  $11!$   $B$ :  $9!$   $C$ :  $10!$   $D$ :  $3^{11}$

**Question 4:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

$A$ :  $20 \cdot 19 \cdot 18$   $B$ :  $3^{20}$   $C$ :  $20^3$   $D$ :  $\frac{20!}{3!}$

**Question 5:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

$A$ :  $\frac{k(k-1)\cdots(k-n+1)}{n \cdot (n-1)\cdots 2 \cdot 1}$   $B$ :  $\frac{n(n-1)\cdots(n-k+1)}{k \cdot (k-1)\cdots 2 \cdot 1}$

**Question 6:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

$A$ :  $10 \cdot 9 \cdot 8$   $B$ :  $30$   $C$ :  $3^{10}$   $D$ :  $10^3$

**Question 7:** The binomial coefficient  $\binom{n}{k}$  equals

$A$ :  $\binom{n}{n-k}$ .  $B$ : 0 if  $k = 0$ .

**Question 8:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

$A$ :  $\frac{10!}{6!}$   $B$ :  $\frac{10!}{6!4!}$   $C$ :  $10^4$   $D$ :  $6!$

**Question 9:** If  $G$  is a simple graph then

$A$ : it has at least two vertices with odd degree.  $B$ : the number of its vertices with even degree is even.  $C$ : it has at most two vertices with odd degree.  $D$ : the number of its vertices with odd degree is not odd.

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $2(m+n)$  B:  $m \cdot n$  C:  $m(n-1) + n(m-1)$  D:  $m+n$

**Question 2:** The binomial coefficient  $\binom{n}{k}$  equals

A: 0 if  $k=0$ . B:  $\binom{n}{n-k}$ .

**Question 3:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: each vertex of side  $A$  is connected with all vertices of side  $B$ . B: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . C: each vertex of side  $B$  is connected to some vertex in side  $A$ . D: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .

**Question 4:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $10!$  B:  $9!$  C:  $11!$  D:  $3^{11}$

**Question 5:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ . B: there is always a perfect matching of the vertices of side  $A$ . C: side  $B$  has more vertices than side  $A$ . D: side  $A$  has more vertices than side  $B$ .

**Question 6:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $2^{10}$  B:  $11!$  C:  $10 \times 10$  D:  $10!$

**Question 7:** How many different quadruples can one form from the objects  $1, 1, 2, 3, 4, 5, 6, 7, 8, 9$ . Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 8:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $3^n$  B:  $2^n + 2^n$  C:  $2^n$  D:  $\binom{n}{n/2}$

**Question 9:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A: 30 B:  $3^{10}$  C:  $10^3$  D:  $10 \cdot 9 \cdot 8$

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Instructor: Mihalis Kolountzakis

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $3^n$  B:  $2^n + 2^n$  C:  $\binom{n}{n/2}$  D:  $2^n$

**Question 2:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $10 \times 10$  B:  $2^{10}$  C:  $11!$  D:  $10!$

**Question 3:** If  $G$  is a simple graph then

A: the number of its vertices with odd degree is not odd. B: the number of its vertices with even degree is even. C: it has at least two vertices with odd degree. D: it has at most two vertices with odd degree.

**Question 4:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{n(n-1)\dots(n-k+1)}{k(k-1)\dots 2 \cdot 1}$  B:  $\frac{k(k-1)\dots(k-n+1)}{n(n-1)\dots 2 \cdot 1}$

**Question 5:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m \cdot n$  B:  $2(m+n)$  C:  $m(n-1) + n(m-1)$  D:  $m+n$

**Question 6:** In a simple graph with 100 vertices

A: the minimum vertex degree is  $\geq 1$ . B: not all vertex degrees can be odd. C: the maximum vertex degree is  $\leq 99$ . D: it is possible that all vertices have different degrees.

**Question 7:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $3^{10}$  B: 30 C:  $10^3$  D:  $10 \cdot 9 \cdot 8$

**Question 8:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 9:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!4!}$  B:  $6!$  C:  $10^4$  D:  $\frac{10!}{6!}$

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Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $m \cdot n$  B:  $n(n-1) \cdots (n-m+1)$  C:  $n^m$  D:  $m^n$

**Question 2:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

**Question 3:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!}$  B:  $6!$  C:  $\frac{10!}{6!4!}$  D:  $10^4$

**Question 4:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $10!$  B:  $3^{11}$  C:  $11!$  D:  $9!$

**Question 5:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m+n$  B:  $m(n-1) + n(m-1)$  C:  $2(m+n)$  D:  $m \cdot n$

**Question 6:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 7:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: each vertex of side  $A$  is connected with all vertices of side  $B$ . B: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . C: each vertex of side  $B$  is connected to some vertex in side  $A$ . D: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .

**Question 8:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $3^{10}$  B:  $10 \cdot 9 \cdot 8$  C: 30 D:  $10^3$

**Question 9:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: there is always a perfect matching of the vertices of side  $A$ . B: side  $B$  has more vertices than side  $A$ . C: side  $A$  has more vertices than side  $B$ . D: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ .

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $3^{11}$  B:  $11!$  C:  $9!$  D:  $10!$

**Question 2:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A: 30 B:  $10 \cdot 9 \cdot 8$  C:  $10^3$  D:  $3^{10}$

**Question 3:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m + n$  B:  $m \cdot n$  C:  $2(m + n)$  D:  $m(n - 1) + n(m - 1)$

**Question 4:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1)\dots(k-n+1)}{n \cdot (n-1) \dots 2 \cdot 1}$  B:  $\frac{n(n-1)\dots(n-k+1)}{k \cdot (k-1) \dots 2 \cdot 1}$

**Question 5:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 6:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $2^n$  B:  $2^n + 2^n$  C:  $\binom{n}{n/2}$  D:  $3^n$

**Question 7:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: each vertex of side  $A$  is connected with all vertices of side  $B$ . B: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . C: each vertex of side  $B$  is connected to some vertex in side  $A$ . D: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .

**Question 8:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $6!$  B:  $10^4$  C:  $\frac{10!}{6!}$  D:  $\frac{10!}{6!4!}$

**Question 9:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it must have at least  $n$  edges. B: it cannot contain cycles. C: it must have at least  $n - 1$  edges. D: it cannot have more than  $n + 1$  edges.

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Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{n(n-1)\cdots(n-k+1)}{k\cdot(k-1)\cdots 2\cdot 1}$     B:  $\frac{k(k-1)\cdots(k-n+1)}{n\cdot(n-1)\cdots 2\cdot 1}$

**Question 2:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $10^4$     B:  $\frac{10!}{6!}$     C:  $6!$     D:  $\frac{10!}{6!4!}$

**Question 3:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .    B: each vertex of side  $A$  is connected with all vertices of side  $B$ .    C: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .    D: each vertex of side  $B$  is connected to some vertex in side  $A$ .

**Question 4:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $m \cdot n$     B:  $m^n$     C:  $n(n-1)\cdots(n-m+1)$     D:  $n^m$

**Question 5:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it cannot have more than  $n+1$  edges.    B: it must have at least  $n-1$  edges.    C: it cannot contain cycles.    D: it must have at least  $n$  edges.

**Question 6:** In a simple graph with 100 vertices

A: the minimum vertex degree is  $\geq 1$ .    B: it is possible that all vertices have different degrees.    C: not all vertex degrees can be odd.    D: the maximum vertex degree is  $\leq 99$ .

**Question 7:** The binomial coefficient  $\binom{n}{k}$  equals

A: 0 if  $k=0$ .    B:  $\binom{n}{n-k}$ .

**Question 8:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $3^{10}$     B: 30    C:  $10 \cdot 9 \cdot 8$     D:  $10^3$

**Question 9:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $11!$     B:  $2^{10}$     C:  $10 \times 10$     D:  $10!$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** If  $G$  is a simple graph then

$A$ : the number of its vertices with odd degree is not odd.  $B$ : it has at least two vertices with odd degree.

$C$ : the number of its vertices with even degree is even.  $D$ : it has at most two vertices with odd degree.

**Question 2:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

$A$ :  $\frac{k(k-1)\cdots(k-n+1)}{n\cdot(n-1)\cdots 2\cdot 1}$   $B$ :  $\frac{n(n-1)\cdots(n-k+1)}{k\cdot(k-1)\cdots 2\cdot 1}$

**Question 3:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

$A$ :  $9!$   $B$ :  $3^{11}$   $C$ :  $10!$   $D$ :  $11!$

**Question 4:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

$A$ :  $10 \cdot 9 \cdot 8$   $B$ :  $30$   $C$ :  $3^{10}$   $D$ :  $10^3$

**Question 5:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

$A$ :  $n^m$   $B$ :  $m \cdot n$   $C$ :  $m^n$   $D$ :  $n(n-1)\cdots(n-m+1)$

**Question 6:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

$A$ : equal to 0 exactly when  $i$  is not connected to  $j$   $B$ : equal to 1 exactly when there is a path that connect  $i$  to  $j$ .  $C$ : equal to the degree of vertex  $i$   $D$ : equal to 1 exactly when  $i$  is not connected to  $j$

**Question 7:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

$A$ : side  $B$  has more vertices than side  $A$ .  $B$ : side  $A$  has more vertices than side  $B$ .  $C$ : For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ .  $D$ : there is always a perfect matching of the vertices of side  $A$ .

**Question 8:** How many different quadruples can one form from the objects  $1, 1, 2, 3, 4, 5, 6, 7, 8, 9$ . Two quadruples differing only in order are not considered different.

$A$ :  $10 \cdot 9 \cdot 8 \cdot 7$   $B$ :  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 9:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

$A$ :  $6!$   $B$ :  $\frac{10!}{6!}$   $C$ :  $\frac{10!}{6!4!}$   $D$ :  $10^4$

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Instructor: Mihalis Kolountzakis

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $3^{11}$  B:  $9!$  C:  $11!$  D:  $10!$

**Question 2:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to 1 exactly when  $i$  is not connected to  $j$  B: equal to 1 exactly when there is a path that connect  $i$  to  $j$ . C: equal to the degree of vertex  $i$  D: equal to 0 exactly when  $i$  is not connected to  $j$

**Question 3:** The binomial coefficient  $\binom{n}{k}$  equals

A: 0 if  $k = 0$ . B:  $\binom{n}{n-k}$ .

**Question 4:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $11!$  B:  $10!$  C:  $2^{10}$  D:  $10 \times 10$

**Question 5:** How many different quadruples can one form from the objects  $1, 1, 2, 3, 4, 5, 6, 7, 8, 9$ . Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 6:** If  $G$  is a simple graph then

A: the number of its vertices with even degree is even. B: it has at least two vertices with odd degree.

C: the number of its vertices with odd degree is not odd. D: it has at most two vertices with odd degree.

**Question 7:** In a simple graph with 100 vertices

A: not all vertex degrees can be odd. B: the maximum vertex degree is  $\leq 99$ . C: it is possible that all vertices have different degrees. D: the minimum vertex degree is  $\geq 1$ .

**Question 8:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $3^n$  B:  $2^n$  C:  $\binom{n}{n/2}$  D:  $2^n + 2^n$

**Question 9:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A: 30 B:  $10 \cdot 9 \cdot 8$  C:  $10^3$  D:  $3^{10}$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $2^{10}$  B:  $11!$  C:  $10 \times 10$  D:  $10!$

**Question 2:** How many different quadruples can one form from the objects  $1, 1, 2, 3, 4, 5, 6, 7, 8, 9$ . Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 3:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $20 \cdot 19 \cdot 18$  B:  $20^3$  C:  $3^{20}$  D:  $\frac{20!}{3!}$

**Question 4:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to 1 exactly when there is a path that connect  $i$  to  $j$ . B: equal to the degree of vertex  $i$  C: equal to 0 exactly when  $i$  is not connected to  $j$  D: equal to 1 exactly when  $i$  is not connected to  $j$

**Question 5:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it cannot have more than  $n + 1$  edges. B: it cannot contain cycles. C: it must have at least  $n$  edges. D: it must have at least  $n - 1$  edges.

**Question 6:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $2(m + n)$  B:  $m(n - 1) + n(m - 1)$  C:  $m + n$  D:  $m \cdot n$

**Question 7:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{n(n-1)\dots(n-k+1)}{k \cdot (k-1) \dots 2 \cdot 1}$  B:  $\frac{k(k-1)\dots(k-n+1)}{n \cdot (n-1) \dots 2 \cdot 1}$

**Question 8:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $n(n - 1) \cdots (n - m + 1)$  B:  $m \cdot n$  C:  $n^m$  D:  $m^n$

**Question 9:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $6!$  B:  $10^4$  C:  $\frac{10!}{6!}$  D:  $\frac{10!}{6!4!}$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

**Question 2:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{n(n-1)\cdots(n-k+1)}{k\cdot(k-1)\cdots 2\cdot 1}$  B:  $\frac{k(k-1)\cdots(k-n+1)}{n\cdot(n-1)\cdots 2\cdot 1}$

**Question 3:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $3^{20}$  B:  $20 \cdot 19 \cdot 18$  C:  $\frac{20!}{3!}$  D:  $20^3$

**Question 4:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $10 \cdot 9 \cdot 8$  B:  $3^{10}$  C:  $10^3$  D: 30

**Question 5:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ . B: side  $B$  has more vertices than side  $A$ . C: side  $A$  has more vertices than side  $B$ . D: there is always a perfect matching of the vertices of side  $A$ .

**Question 6:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $10!$  B:  $3^{11}$  C:  $9!$  D:  $11!$

**Question 7:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $10^4$  B:  $\frac{10!}{6!}$  C:  $6!$  D:  $\frac{10!}{6!4!}$

**Question 8:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . B: each vertex of side  $B$  is connected to some vertex in side  $A$ . C: each vertex of side  $A$  is connected with all vertices of side  $B$ . D: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .

**Question 9:** If  $G$  is a simple graph then

A: the number of its vertices with even degree is even. B: it has at most two vertices with odd degree. C: it has at least two vertices with odd degree. D: the number of its vertices with odd degree is not odd.

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$   
 $A$ : the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .  $B$ : each vertex of side  $B$  is connected to some vertex in side  $A$ .  $C$ : each vertex of side  $A$  is connected with all vertices of side  $B$ .  $D$ : the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .

**Question 2:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

$A$ :  $n(n-1) \cdots (n-m+1)$   $B$ :  $m \cdot n$   $C$ :  $m^n$   $D$ :  $n^m$

**Question 3:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

$A$ :  $10 \cdot 9 \cdot 8 \cdot 7$   $B$ :  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 4:** How many circular orderings of the numbers 0, 1, ..., 10 are there? (Two circular orderings which differ only by a rotation are not considered different.)

$A$ :  $3^{11}$   $B$ :  $9!$   $C$ :  $10!$   $D$ :  $11!$

**Question 5:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

$A$ : side  $A$  has more vertices than side  $B$ .  $B$ : For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ .  $C$ : there is always a perfect matching of the vertices of side  $A$ .  $D$ : side  $B$  has more vertices than side  $A$ .

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$A$ :  $\frac{10!}{6!}$   $B$ :  $10^4$   $C$ :  $\frac{10!}{6!4!}$   $D$ :  $6!$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

$A$ : there is always a perfect matching of the vertices of side  $A$ .  $B$ : side  $A$  has more vertices than side  $B$ .  $C$ : side  $B$  has more vertices than side  $A$ .  $D$ : For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ .

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$A$ : it must have at least  $n - 1$  edges.  $B$ : it cannot contain cycles.  $C$ : it cannot have more than  $n + 1$  edges.  $D$ : it must have at least  $n$  edges.

**Question 6:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

$A$ :  $2^n$   $B$ :  $2^n + 2^n$   $C$ :  $\binom{n}{n/2}$   $D$ :  $3^n$

**Question 7:** If  $G$  is a simple graph then

$A$ : the number of its vertices with even degree is even.  $B$ : the number of its vertices with odd degree is not odd.  $C$ : it has at most two vertices with odd degree.  $D$ : it has at least two vertices with odd degree.

**Question 8:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

$A$ :  $10!$   $B$ :  $11!$   $C$ :  $3^{11}$   $D$ :  $9!$

**Question 9:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

$A$ :  $11!$   $B$ :  $2^{10}$   $C$ :  $10 \times 10$   $D$ :  $10!$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** If  $G$  is a simple graph then

$A$ : the number of its vertices with even degree is even.  $B$ : it has at most two vertices with odd degree.  
 $C$ : it has at least two vertices with odd degree.  $D$ : the number of its vertices with odd degree is not odd.

**Question 2:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

$A$ :  $6!$   $B$ :  $\frac{10!}{6!}$   $C$ :  $\frac{10!}{6!4!}$   $D$ :  $10^4$

**Question 3:** If  $G$  is a connected simple graph with  $n$  vertices then

$A$ : it cannot contain cycles.  $B$ : it cannot have more than  $n + 1$  edges.  $C$ : it must have at least  $n$  edges.  
 $D$ : it must have at least  $n - 1$  edges.

**Question 4:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

$A$ :  $10 \cdot 9 \cdot 8 \cdot 7$   $B$ :  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 5:** The binomial coefficient  $\binom{n}{k}$  equals

$A$ : 0 if  $k = 0$ .  $B$ :  $\binom{n}{n-k}$ .

**Question 6:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

$A$ :  $2(m + n)$   $B$ :  $m(n - 1) + n(m - 1)$   $C$ :  $m \cdot n$   $D$ :  $m + n$

**Question 7:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

$A$ :  $2^n + 2^n$   $B$ :  $3^n$   $C$ :  $2^n$   $D$ :  $\binom{n}{n/2}$

**Question 8:** In how many ways can the numbers 0, 1,  $\dots$ , 10 be put in order?

$A$ :  $2^{10}$   $B$ :  $10!$   $C$ :  $10 \times 10$   $D$ :  $11!$

**Question 9:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

$A$ :  $m \cdot n$   $B$ :  $n^m$   $C$ :  $m^n$   $D$ :  $n(n - 1) \cdots (n - m + 1)$

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Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $3^n$  B:  $2^n$  C:  $2^n + 2^n$  D:  $\binom{n}{n/2}$

**Question 2:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 3:** In how many ways can the numbers 0, 1,  $\dots$ , 10 be put in order?

A:  $10 \times 10$  B:  $10!$  C:  $11!$  D:  $2^{10}$

**Question 4:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: there is always a perfect matching of the vertices of side  $A$ . B: side  $A$  has more vertices than side  $B$ . C: side  $B$  has more vertices than side  $A$ . D: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ .

**Question 5:** The binomial coefficient  $\binom{n}{k}$  equals

A: 0 if  $k = 0$ . B:  $\binom{n}{n-k}$ .

**Question 6:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!4!}$  B:  $6!$  C:  $10^4$  D:  $\frac{10!}{6!}$

**Question 7:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it cannot contain cycles. B: it must have at least  $n - 1$  edges. C: it cannot have more than  $n + 1$  edges. D: it must have at least  $n$  edges.

**Question 8:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $2(m + n)$  B:  $m(n - 1) + n(m - 1)$  C:  $m \cdot n$  D:  $m + n$

**Question 9:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $3^{10}$  B: 30 C:  $10 \cdot 9 \cdot 8$  D:  $10^3$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $20^3$  B:  $\frac{20!}{3!}$  C:  $20 \cdot 19 \cdot 18$  D:  $3^{20}$

**Question 2:** If  $G$  is a simple graph then

A: it has at least two vertices with odd degree. B: the number of its vertices with even degree is even. C: the number of its vertices with odd degree is not odd. D: it has at most two vertices with odd degree.

**Question 3:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: side  $B$  has more vertices than side  $A$ . B: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ . C: there is always a perfect matching of the vertices of side  $A$ . D: side  $A$  has more vertices than side  $B$ .

**Question 4:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 5:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $6!$  B:  $\frac{10!}{6!}$  C:  $\frac{10!}{6!4!}$  D:  $10^4$

**Question 6:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it must have at least  $n - 1$  edges. B: it must have at least  $n$  edges. C: it cannot contain cycles. D: it cannot have more than  $n + 1$  edges.

**Question 7:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $2^n + 2^n$  B:  $2^n$  C:  $\binom{n}{n/2}$  D:  $3^n$

**Question 8:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $9!$  B:  $3^{11}$  C:  $11!$  D:  $10!$

**Question 9:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1)\cdots(k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$  B:  $\frac{n(n-1)\cdots(n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $2^n$  B:  $3^n$  C:  $2^n + 2^n$  D:  $\binom{n}{n/2}$

**Question 2:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $2(m+n)$  B:  $m+n$  C:  $m(n-1) + n(m-1)$  D:  $m \cdot n$

**Question 3:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $11!$  B:  $10!$  C:  $2^{10}$  D:  $10 \times 10$

**Question 4:** How many different quadruples can one form from the objects  $1, 1, 2, 3, 4, 5, 6, 7, 8, 9$ . Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 5:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

**Question 6:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it must have at least  $n - 1$  edges. B: it must have at least  $n$  edges. C: it cannot have more than  $n + 1$  edges. D: it cannot contain cycles.

**Question 7:** If  $G$  is a simple graph then

A: the number of its vertices with even degree is even. B: the number of its vertices with odd degree is not odd. C: it has at most two vertices with odd degree. D: it has at least two vertices with odd degree.

**Question 8:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $10^3$  B:  $3^{10}$  C: 30 D:  $10 \cdot 9 \cdot 8$

**Question 9:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!4!}$  B:  $10^4$  C:  $\frac{10!}{6!}$  D:  $6!$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $9!$  B:  $3^{11}$  C:  $11!$  D:  $10!$

**Question 2:** How many different quadruples can one form from the objects  $1, 1, 2, 3, 4, 5, 6, 7, 8, 9$ . Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 3:** The binomial coefficient  $\binom{n}{k}$  equals

A: 0 if  $k = 0$ . B:  $\binom{n}{n-k}$ .

**Question 4:** If  $G$  is a simple graph then

A: it has at least two vertices with odd degree. B: the number of its vertices with odd degree is not odd. C: the number of its vertices with even degree is even. D: it has at most two vertices with odd degree.

**Question 5:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ . B: there is always a perfect matching of the vertices of side  $A$ . C: side  $A$  has more vertices than side  $B$ . D: side  $B$  has more vertices than side  $A$ .

**Question 6:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $3^{10}$  B:  $10 \cdot 9 \cdot 8$  C: 30 D:  $10^3$

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**Question 8:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $n^m$  B:  $m \cdot n$  C:  $n(n-1) \cdots (n-m+1)$  D:  $m^n$

**Question 9:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: each vertex of side  $A$  is connected with all vertices of side  $B$ . B: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . C: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . D: each vertex of side  $B$  is connected to some vertex in side  $A$ .

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $11!$  B:  $3^{11}$  C:  $10!$  D:  $9!$

**Question 2:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to 1 exactly when there is a path that connect  $i$  to  $j$ . B: equal to 0 exactly when  $i$  is not connected to  $j$  C: equal to 1 exactly when  $i$  is not connected to  $j$  D: equal to the degree of vertex  $i$

**Question 3:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

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A:  $10 \cdot 9 \cdot 8$  B:  $10^3$  C: 30 D:  $3^{10}$

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**Question 6:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: each vertex of side  $B$  is connected to some vertex in side  $A$ . B: each vertex of side  $A$  is connected with all vertices of side  $B$ . C: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . D: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .

**Question 7:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $\binom{n}{n/2}$  B:  $3^n$  C:  $2^n$  D:  $2^n + 2^n$

**Question 8:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $10 \times 10$  B:  $11!$  C:  $10!$  D:  $2^{10}$

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A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $10!$  B:  $3^{11}$  C:  $11!$  D:  $9!$

**Question 2:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: side  $B$  has more vertices than side  $A$ . B: side  $A$  has more vertices than side  $B$ . C: there is always a perfect matching of the vertices of side  $A$ . D: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ .

**Question 3:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $20 \cdot 19 \cdot 18$  B:  $20^3$  C:  $3^{20}$  D:  $\frac{20!}{3!}$

**Question 4:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it must have at least  $n - 1$  edges. B: it cannot have more than  $n + 1$  edges. C: it must have at least  $n$  edges. D: it cannot contain cycles.

**Question 5:** The binomial coefficient  $\binom{n}{k}$  equals

A: 0 if  $k = 0$ . B:  $\binom{n}{n-k}$ .

**Question 6:** In a simple graph with 100 vertices

A: the maximum vertex degree is  $\leq 99$ . B: the minimum vertex degree is  $\geq 1$ . C: it is possible that all vertices have different degrees. D: not all vertex degrees can be odd.

**Question 7:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $10^4$  B:  $\frac{10!}{6!4!}$  C:  $6!$  D:  $\frac{10!}{6!}$

**Question 8:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1)\dots(k-n+1)}{n(n-1)\dots 2 \cdot 1}$  B:  $\frac{n(n-1)\dots(n-k+1)}{k(k-1)\dots 2 \cdot 1}$

**Question 9:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $2^n + 2^n$  B:  $2^n$  C:  $\binom{n}{n/2}$  D:  $3^n$

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Name:

UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$   
 $A$ : the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .  $B$ : each vertex of side  $B$  is connected to some vertex in side  $A$ .  $C$ : each vertex of side  $A$  is connected with all vertices of side  $B$ .  $D$ : the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .

**Question 2:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

$A$ :  $10^4$   $B$ :  $6!$   $C$ :  $\frac{10!}{6!}$   $D$ :  $\frac{10!}{6!4!}$

**Question 3:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

$A$ :  $n(n-1) \cdots (n-m+1)$   $B$ :  $m^n$   $C$ :  $n^m$   $D$ :  $m \cdot n$

**Question 4:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

$A$ :  $\frac{k(k-1) \cdots (k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$   $B$ :  $\frac{n(n-1) \cdots (n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$

**Question 5:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

$A$ : equal to 0 exactly when  $i$  is not connected to  $j$   $B$ : equal to 1 exactly when  $i$  is not connected to  $j$

$C$ : equal to the degree of vertex  $i$   $D$ : equal to 1 exactly when there is a path that connect  $i$  to  $j$ .

**Question 6:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

$A$ :  $10^3$   $B$ :  $3^{10}$   $C$ : 30  $D$ :  $10 \cdot 9 \cdot 8$

**Question 7:** If  $G$  is a simple graph then

$A$ : the number of its vertices with odd degree is not odd.  $B$ : it has at most two vertices with odd degree.

$C$ : the number of its vertices with even degree is even.  $D$ : it has at least two vertices with odd degree.

**Question 8:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

$A$ :  $11!$   $B$ :  $3^{11}$   $C$ :  $9!$   $D$ :  $10!$

**Question 9:** The binomial coefficient  $\binom{n}{k}$  equals

$A$ :  $\binom{n}{n-k}$ .  $B$ : 0 if  $k = 0$ .

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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Name:

UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

$A$ : equal to 0 exactly when  $i$  is not connected to  $j$     $B$ : equal to 1 exactly when  $i$  is not connected to  $j$

$C$ : equal to 1 exactly when there is a path that connect  $i$  to  $j$ .    $D$ : equal to the degree of vertex  $i$

**Question 2:** If  $G$  is a connected simple graph with  $n$  vertices then

$A$ : it cannot have more than  $n + 1$  edges.    $B$ : it must have at least  $n$  edges.    $C$ : it cannot contain cycles.

$D$ : it must have at least  $n - 1$  edges.

**Question 3:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

$A$ : side  $A$  has more vertices than side  $B$ .    $B$ : For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ .    $C$ : there is always a perfect matching of the vertices of side  $A$ .    $D$ : side  $B$  has more vertices than side  $A$ .

**Question 4:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

$A$ :  $3^{11}$     $B$ :  $9!$     $C$ :  $10!$     $D$ :  $11!$

**Question 5:** How many different quadruples can one form from the objects  $1, 1, 2, 3, 4, 5, 6, 7, 8, 9$ . Two quadruples differing only in order are not considered different.

$A$ :  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$     $B$ :  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 6:** The binomial coefficient  $\binom{n}{k}$  equals

$A$ :  $\binom{n}{n-k}$ .    $B$ : 0 if  $k = 0$ .

**Question 7:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

$A$ :  $2^n + 2^n$     $B$ :  $\binom{n}{n/2}$     $C$ :  $3^n$     $D$ :  $2^n$

**Question 8:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

$A$ :  $10 \times 10$     $B$ :  $11!$     $C$ :  $10!$     $D$ :  $2^{10}$

**Question 9:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

$A$ : 30    $B$ :  $3^{10}$     $C$ :  $10 \cdot 9 \cdot 8$     $D$ :  $10^3$

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Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $6!$  B:  $10^4$  C:  $\frac{10!}{6!}$  D:  $\frac{10!}{6!4!}$

**Question 2:** In a simple graph with 100 vertices

A: it is possible that all vertices have different degrees. B: the minimum vertex degree is  $\geq 1$ . C: not all vertex degrees can be odd. D: the maximum vertex degree is  $\leq 99$ .

**Question 3:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 4:** In how many ways can the numbers 0, 1,  $\dots$ , 10 be put in order?

A:  $11!$  B:  $10!$  C:  $10 \times 10$  D:  $2^{10}$

**Question 5:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it cannot contain cycles. B: it must have at least  $n - 1$  edges. C: it must have at least  $n$  edges. D: it cannot have more than  $n + 1$  edges.

**Question 6:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $n(n - 1) \cdots (n - m + 1)$  B:  $m^n$  C:  $m \cdot n$  D:  $n^m$

**Question 7:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to 1 exactly when there is a path that connect  $i$  to  $j$ . B: equal to 1 exactly when  $i$  is not connected to  $j$  C: equal to the degree of vertex  $i$  D: equal to 0 exactly when  $i$  is not connected to  $j$

**Question 8:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1)\cdots(k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$  B:  $\frac{n(n-1)\cdots(n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$

**Question 9:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $20^3$  B:  $20 \cdot 19 \cdot 18$  C:  $\frac{20!}{3!}$  D:  $3^{20}$

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Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m \cdot n$  B:  $m(n-1) + n(m-1)$  C:  $m+n$  D:  $2(m+n)$

**Question 2:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $n(n-1) \cdots (n-m+1)$  B:  $m^n$  C:  $n^m$  D:  $m \cdot n$

**Question 3:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $10!$  B:  $2^{10}$  C:  $11!$  D:  $10 \times 10$

**Question 4:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $\binom{n}{n/2}$  B:  $3^n$  C:  $2^n + 2^n$  D:  $2^n$

**Question 5:** The binomial coefficient  $\binom{n}{k}$  equals

A: 0 if  $k = 0$ . B:  $\binom{n}{n-k}$ .

**Question 6:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it must have at least  $n-1$  edges. B: it must have at least  $n$  edges. C: it cannot contain cycles. D: it cannot have more than  $n+1$  edges.

**Question 7:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: each vertex of side  $A$  is connected with all vertices of side  $B$ . B: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . C: each vertex of side  $B$  is connected to some vertex in side  $A$ . D: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .

**Question 8:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1) \cdots (k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$  B:  $\frac{n(n-1) \cdots (n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$

**Question 9:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $6!$  B:  $\frac{10!}{6!}$  C:  $\frac{10!}{6!4!}$  D:  $10^4$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $10^4$  B:  $6!$  C:  $\frac{10!}{6!4!}$  D:  $\frac{10!}{6!}$

**Question 2:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . B: each vertex of side  $A$  is connected with all vertices of side  $B$ . C: each vertex of side  $B$  is connected to some vertex in side  $A$ . D: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .

**Question 3:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1)\dots(k-n+1)}{n(n-1)\dots 2 \cdot 1}$  B:  $\frac{n(n-1)\dots(n-k+1)}{k(k-1)\dots 2 \cdot 1}$

**Question 4:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $2(m+n)$  B:  $m(n-1) + n(m-1)$  C:  $m \cdot n$  D:  $m+n$

**Question 5:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

**Question 6:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $\frac{20!}{3!}$  B:  $3^{20}$  C:  $20 \cdot 19 \cdot 18$  D:  $20^3$

**Question 7:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $3^n$  B:  $2^n + 2^n$  C:  $2^n$  D:  $\binom{n}{n/2}$

**Question 8:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $10!$  B:  $11!$  C:  $9!$  D:  $3^{11}$

**Question 9:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: side  $A$  has more vertices than side  $B$ . B: side  $B$  has more vertices than side  $A$ . C: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ . D: there is always a perfect matching of the vertices of side  $A$ .

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$   
 $A$ : each vertex of side  $B$  is connected to some vertex in side  $A$ .  $B$ : the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .  $C$ : each vertex of side  $A$  is connected with all vertices of side  $B$ .  $D$ : the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .

**Question 2:** The binomial coefficient  $\binom{n}{k}$  equals

$A$ :  $\binom{n}{n-k}$ .  $B$ : 0 if  $k = 0$ .

**Question 3:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

$A$ :  $10 \cdot 9 \cdot 8 \cdot 7$   $B$ :  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 4:** If  $G$  is a simple graph then

$A$ : the number of its vertices with even degree is even.  $B$ : the number of its vertices with odd degree is not odd.  $C$ : it has at most two vertices with odd degree.  $D$ : it has at least two vertices with odd degree.

**Question 5:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

$A$ :  $\frac{20!}{3!}$   $B$ :  $20 \cdot 19 \cdot 18$   $C$ :  $20^3$   $D$ :  $3^{20}$

**Question 6:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

$A$ :  $11!$   $B$ :  $3^{11}$   $C$ :  $10!$   $D$ :  $9!$

**Question 7:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

$A$ : equal to the degree of vertex  $i$   $B$ : equal to 0 exactly when  $i$  is not connected to  $j$   $C$ : equal to 1 exactly when  $i$  is not connected to  $j$   $D$ : equal to 1 exactly when there is a path that connect  $i$  to  $j$ .

**Question 8:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

$A$ :  $2^{10}$   $B$ :  $10 \times 10$   $C$ :  $11!$   $D$ :  $10!$

**Question 9:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

$A$ :  $n^m$   $B$ :  $n(n-1) \cdots (n-m+1)$   $C$ :  $m^n$   $D$ :  $m \cdot n$

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Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $2^n + 2^n$  B:  $2^n$  C:  $\binom{n}{n/2}$  D:  $3^n$

**Question 2:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $3^{11}$  B:  $9!$  C:  $10!$  D:  $11!$

**Question 3:** If  $G$  is a simple graph then

A: it has at most two vertices with odd degree. B: the number of its vertices with odd degree is not odd.

C: it has at least two vertices with odd degree. D: the number of its vertices with even degree is even.

**Question 4:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it cannot contain cycles. B: it cannot have more than  $n + 1$  edges. C: it must have at least  $n - 1$  edges. D: it must have at least  $n$  edges.

**Question 5:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $3^{20}$  B:  $20 \cdot 19 \cdot 18$  C:  $20^3$  D:  $\frac{20!}{3!}$

**Question 6:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $6!$  B:  $10^4$  C:  $\frac{10!}{6!4!}$  D:  $\frac{10!}{6!}$

**Question 7:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1)\cdots(k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$  B:  $\frac{n(n-1)\cdots(n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$

**Question 8:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: each vertex of side  $A$  is connected with all vertices of side  $B$ . B: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . C: each vertex of side  $B$  is connected to some vertex in side  $A$ .

D: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .

**Question 9:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In a simple graph with 100 vertices

*A:* not all vertex degrees can be odd. *B:* the minimum vertex degree is  $\geq 1$ . *C:* it is possible that all vertices have different degrees. *D:* the maximum vertex degree is  $\leq 99$ .

**Question 2:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

*A:*  $10^3$  *B:* 30 *C:*  $3^{10}$  *D:*  $10 \cdot 9 \cdot 8$

**Question 3:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

*A:* the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . *B:* each vertex of side  $B$  is connected to some vertex in side  $A$ . *C:* each vertex of side  $A$  is connected with all vertices of side  $B$ . *D:* the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .

**Question 4:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

*A:*  $10!$  *B:*  $11!$  *C:*  $3^{11}$  *D:*  $9!$

**Question 5:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

*A:*  $\frac{n(n-1)\dots(n-k+1)}{k \cdot (k-1) \dots 2 \cdot 1}$  *B:*  $\frac{k(k-1)\dots(k-n+1)}{n \cdot (n-1) \dots 2 \cdot 1}$

**Question 6:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

*A:*  $2^n$  *B:*  $\binom{n}{n/2}$  *C:*  $2^n + 2^n$  *D:*  $3^n$

**Question 7:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

*A:*  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  *B:*  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 8:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

*A:*  $\frac{10!}{6!4!}$  *B:*  $\frac{10!}{6!}$  *C:*  $6!$  *D:*  $10^4$

**Question 9:** If  $G$  is a connected simple graph with  $n$  vertices then

*A:* it cannot contain cycles. *B:* it must have at least  $n - 1$  edges. *C:* it must have at least  $n$  edges. *D:* it cannot have more than  $n + 1$  edges.

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Instructor: Mihalis Kolountzakis

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m(n-1) + n(m-1)$  B:  $m+n$  C:  $m \cdot n$  D:  $2(m+n)$

**Question 2:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to 0 exactly when  $i$  is not connected to  $j$  B: equal to the degree of vertex  $i$  C: equal to 1 exactly when  $i$  is not connected to  $j$  D: equal to 1 exactly when there is a path that connect  $i$  to  $j$ .

**Question 3:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $10^3$  B:  $3^{10}$  C:  $10 \cdot 9 \cdot 8$  D: 30

**Question 4:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: each vertex of side  $A$  is connected with all vertices of side  $B$ . B: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . C: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . D: each vertex of side  $B$  is connected to some vertex in side  $A$ .

**Question 5:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $n(n-1) \cdots (n-m+1)$  B:  $m \cdot n$  C:  $n^m$  D:  $m^n$

**Question 6:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

**Question 7:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 8:** In how many ways can the numbers 0, 1, ..., 10 be put in order?

A:  $10 \times 10$  B: 11! C: 10! D:  $2^{10}$

**Question 9:** How many circular orderings of the numbers 0, 1, ..., 10 are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $3^{11}$  B: 11! C: 9! D: 10!

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $11!$  B:  $10!$  C:  $9!$  D:  $3^{11}$

**Question 2:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $10 \cdot 9 \cdot 8$  B:  $30$  C:  $3^{10}$  D:  $10^3$

**Question 3:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{n(n-1)\cdots(n-k+1)}{k \cdot (k-1)\cdots 2 \cdot 1}$  B:  $\frac{k(k-1)\cdots(k-n+1)}{n \cdot (n-1)\cdots 2 \cdot 1}$

**Question 4:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $6!$  B:  $10^4$  C:  $\frac{10!}{6!}$  D:  $\frac{10!}{6!4!}$

**Question 5:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 6:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $n(n-1)\cdots(n-m+1)$  B:  $m^n$  C:  $m \cdot n$  D:  $n^m$

**Question 7:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . B: each vertex of side  $B$  is connected to some vertex in side  $A$ . C: each vertex of side  $A$  is connected with all vertices of side  $B$ . D: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .

**Question 8:** In a simple graph with 100 vertices

A: it is possible that all vertices have different degrees. B: the maximum vertex degree is  $\leq 99$ . C: the minimum vertex degree is  $\geq 1$ . D: not all vertex degrees can be odd.

**Question 9:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: side  $A$  has more vertices than side  $B$ . B: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ . C: side  $B$  has more vertices than side  $A$ . D: there is always a perfect matching of the vertices of side  $A$ .

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Instructor: Mihalis Kolountzakis

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $n^m$  B:  $n(n-1) \cdots (n-m+1)$  C:  $m \cdot n$  D:  $m^n$

**Question 2:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $10!$  B:  $3^{11}$  C:  $9!$  D:  $11!$

**Question 3:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $3^n$  B:  $\binom{n}{n/2}$  C:  $2^n$  D:  $2^n + 2^n$

**Question 4:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!4!}$  B:  $10^4$  C:  $6!$  D:  $\frac{10!}{6!}$

**Question 5:** In a simple graph with 100 vertices

A: the minimum vertex degree is  $\geq 1$ . B: the maximum vertex degree is  $\leq 99$ . C: not all vertex degrees can be odd. D: it is possible that all vertices have different degrees.

**Question 6:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: there is always a perfect matching of the vertices of side  $A$ . B: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ . C: side  $B$  has more vertices than side  $A$ . D: side  $A$  has more vertices than side  $B$ .

**Question 7:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m \cdot n$  B:  $m(n-1) + n(m-1)$  C:  $2(m+n)$  D:  $m+n$

**Question 8:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1) \cdots (k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$  B:  $\frac{n(n-1) \cdots (n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$

**Question 9:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

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Instructor: Mihalis Kolountzakis

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

$$A: \frac{n(n-1)\cdots(n-k+1)}{k\cdot(k-1)\cdots 2\cdot 1} \quad B: \frac{k(k-1)\cdots(k-n+1)}{n\cdot(n-1)\cdots 2\cdot 1}$$

**Question 2:** If  $G$  is a connected simple graph with  $n$  vertices then

$A$ : it cannot have more than  $n + 1$  edges.  $B$ : it cannot contain cycles.  $C$ : it must have at least  $n - 1$  edges.  $D$ : it must have at least  $n$  edges.

**Question 3:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

$$A: \binom{8}{4} + \binom{8}{3} + \binom{8}{2} \quad B: 10 \cdot 9 \cdot 8 \cdot 7$$

**Question 4:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

$$A: 10 \times 10 \quad B: 2^{10} \quad C: 10! \quad D: 11!$$

**Question 5:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

$$A: m + n \quad B: m(n - 1) + n(m - 1) \quad C: m \cdot n \quad D: 2(m + n)$$

**Question 6:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

$$A: \frac{10!}{6!4!} \quad B: \frac{10!}{6!} \quad C: 6! \quad D: 10^4$$

**Question 7:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

$$A: 3^n \quad B: \binom{n}{n/2} \quad C: 2^n \quad D: 2^n + 2^n$$

**Question 8:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

$$A: 20^3 \quad B: \frac{20!}{3!} \quad C: 3^{20} \quad D: 20 \cdot 19 \cdot 18$$

**Question 9:** If  $G$  is a simple graph then

$A$ : it has at least two vertices with odd degree.  $B$ : it has at most two vertices with odd degree.  $C$ : the number of its vertices with odd degree is not odd.  $D$ : the number of its vertices with even degree is even.

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Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** If  $G$  is a connected simple graph with  $n$  vertices then

$A$ : it must have at least  $n - 1$  edges.  $B$ : it must have at least  $n$  edges.  $C$ : it cannot contain cycles.

$D$ : it cannot have more than  $n + 1$  edges.

**Question 2:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

$A$ :  $2^{10}$   $B$ :  $10!$   $C$ :  $10 \times 10$   $D$ :  $11!$

**Question 3:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

$A$ :  $10 \cdot 9 \cdot 8$   $B$ :  $10^3$   $C$ :  $3^{10}$   $D$ :  $30$

**Question 4:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

$A$ : equal to 1 exactly when  $i$  is not connected to  $j$   $B$ : equal to the degree of vertex  $i$   $C$ : equal to 0 exactly when  $i$  is not connected to  $j$   $D$ : equal to 1 exactly when there is a path that connect  $i$  to  $j$ .

**Question 5:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

$A$ : each vertex of side  $B$  is connected to some vertex in side  $A$ .  $B$ : the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .  $C$ : each vertex of side  $A$  is connected with all vertices of side  $B$ .  $D$ : the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .

**Question 6:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

$A$ :  $9!$   $B$ :  $3^{11}$   $C$ :  $11!$   $D$ :  $10!$

**Question 7:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

$A$ :  $3^n$   $B$ :  $2^n$   $C$ :  $2^n + 2^n$   $D$ :  $\binom{n}{n/2}$

**Question 8:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

$A$ :  $\frac{k(k-1)\dots(k-n+1)}{n(n-1)\dots 2 \cdot 1}$   $B$ :  $\frac{n(n-1)\dots(n-k+1)}{k(k-1)\dots 2 \cdot 1}$

**Question 9:** The binomial coefficient  $\binom{n}{k}$  equals

$A$ :  $\binom{n}{n-k}$ .  $B$ : 0 if  $k = 0$ .

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

$A$ : equal to 0 exactly when  $i$  is not connected to  $j$      $B$ : equal to 1 exactly when  $i$  is not connected to  $j$

$C$ : equal to 1 exactly when there is a path that connect  $i$  to  $j$ .     $D$ : equal to the degree of vertex  $i$

**Question 2:** The binomial coefficient  $\binom{n}{k}$  equals

$A$ : 0 if  $k = 0$ .     $B$ :  $\binom{n}{n-k}$ .

**Question 3:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

$A$ :  $\frac{20!}{3!}$      $B$ :  $3^{20}$      $C$ :  $20 \cdot 19 \cdot 18$      $D$ :  $20^3$

**Question 4:** If  $G$  is a connected simple graph with  $n$  vertices then

$A$ : it must have at least  $n$  edges.     $B$ : it cannot contain cycles.     $C$ : it cannot have more than  $n + 1$  edges.

$D$ : it must have at least  $n - 1$  edges.

**Question 5:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

$A$ :  $10!$      $B$ :  $9!$      $C$ :  $11!$      $D$ :  $3^{11}$

**Question 6:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

$A$ :  $6!$      $B$ :  $10^4$      $C$ :  $\frac{10!}{6!4!}$      $D$ :  $\frac{10!}{6!}$

**Question 7:** If  $G$  is a simple graph then

$A$ : it has at least two vertices with odd degree.     $B$ : the number of its vertices with odd degree is not odd.

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**Question 8:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

$A$ :  $10 \cdot 9 \cdot 8 \cdot 7$      $B$ :  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 9:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

$A$ :  $3^{10}$      $B$ :  $10^3$      $C$ :  $10 \cdot 9 \cdot 8$      $D$ : 30

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

$A$ : equal to the degree of vertex  $i$     $B$ : equal to 0 exactly when  $i$  is not connected to  $j$     $C$ : equal to 1 exactly when there is a path that connect  $i$  to  $j$ .    $D$ : equal to 1 exactly when  $i$  is not connected to  $j$

**Question 2:** The binomial coefficient  $\binom{n}{k}$  equals

$A$ : 0 if  $k = 0$ .    $B$ :  $\binom{n}{n-k}$ .

**Question 3:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

$A$ :  $\frac{k(k-1)\cdots(k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$     $B$ :  $\frac{n(n-1)\cdots(n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$

**Question 4:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

$A$ : For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ .    $B$ : there is always a perfect matching of the vertices of side  $A$ .    $C$ : side  $B$  has more vertices than side  $A$ .    $D$ : side  $A$  has more vertices than side  $B$ .

**Question 5:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

$A$ :  $10!$     $B$ :  $10 \times 10$     $C$ :  $11!$     $D$ :  $2^{10}$

**Question 6:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

$A$ :  $10 \cdot 9 \cdot 8$     $B$ :  $10^3$     $C$ :  $3^{10}$     $D$ : 30

**Question 7:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

$A$ :  $m \cdot n$     $B$ :  $n(n-1)\cdots(n-m+1)$     $C$ :  $n^m$     $D$ :  $m^n$

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**Question 9:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

$A$ :  $6!$     $B$ :  $10^4$     $C$ :  $\frac{10!}{6!}$     $D$ :  $\frac{10!}{6!4!}$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

$$A: \frac{k(k-1)\cdots(k-n+1)}{n\cdot(n-1)\cdots 2\cdot 1} \quad B: \frac{n(n-1)\cdots(n-k+1)}{k\cdot(k-1)\cdots 2\cdot 1}$$

**Question 2:** In a simple graph with 100 vertices

$A$ : the maximum vertex degree is  $\leq 99$ .  $B$ : it is possible that all vertices have different degrees.  $C$ : not all vertex degrees can be odd.  $D$ : the minimum vertex degree is  $\geq 1$ .

**Question 3:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

$$A: \frac{20!}{3!} \quad B: 20 \cdot 19 \cdot 18 \quad C: 20^3 \quad D: 3^{20}$$

**Question 4:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

$$A: m \cdot n \quad B: m(n-1) + n(m-1) \quad C: m+n \quad D: 2(m+n)$$

**Question 5:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

$$A: 3^{11} \quad B: 10! \quad C: 9! \quad D: 11!$$

**Question 6:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

$$A: \binom{n}{n/2} \quad B: 2^n \quad C: 2^n + 2^n \quad D: 3^n$$

**Question 7:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

$$A: \binom{8}{4} + \binom{8}{3} + \binom{8}{2} \quad B: 10 \cdot 9 \cdot 8 \cdot 7$$

**Question 8:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

$$A: 10 \times 10 \quad B: 10! \quad C: 11! \quad D: 2^{10}$$

**Question 9:** If  $G$  is a connected simple graph with  $n$  vertices then

$A$ : it must have at least  $n$  edges.  $B$ : it must have at least  $n - 1$  edges.  $C$ : it cannot have more than  $n + 1$  edges.  $D$ : it cannot contain cycles.

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** The binomial coefficient  $\binom{n}{k}$  equals

A: 0 if  $k = 0$ . B:  $\binom{n}{n-k}$ .

**Question 2:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to 0 exactly when  $i$  is not connected to  $j$  B: equal to the degree of vertex  $i$  C: equal to 1 exactly when  $i$  is not connected to  $j$  D: equal to 1 exactly when there is a path that connect  $i$  to  $j$ .

**Question 3:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $\binom{n}{n/2}$  B:  $2^n + 2^n$  C:  $3^n$  D:  $2^n$

**Question 4:** If  $G$  is a simple graph then

A: it has at most two vertices with odd degree. B: the number of its vertices with even degree is even. C: it has at least two vertices with odd degree. D: the number of its vertices with odd degree is not odd.

**Question 5:** In a simple graph with 100 vertices

A: the minimum vertex degree is  $\geq 1$ . B: it is possible that all vertices have different degrees. C: the maximum vertex degree is  $\leq 99$ . D: not all vertex degrees can be odd.

**Question 6:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $6!$  B:  $10^4$  C:  $\frac{10!}{6!}$  D:  $\frac{10!}{6!4!}$

**Question 7:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $2^{10}$  B:  $10!$  C:  $10 \times 10$  D:  $11!$

**Question 8:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1)\dots(k-n+1)}{n \cdot (n-1) \dots 2 \cdot 1}$  B:  $\frac{n(n-1)\dots(n-k+1)}{k \cdot (k-1) \dots 2 \cdot 1}$

**Question 9:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $20^3$  B:  $3^{20}$  C:  $20 \cdot 19 \cdot 18$  D:  $\frac{20!}{3!}$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $3^{11}$  B:  $9!$  C:  $10!$  D:  $11!$

**Question 2:** If  $G$  is a simple graph then

A: it has at least two vertices with odd degree. B: the number of its vertices with odd degree is not odd.

C: the number of its vertices with even degree is even. D: it has at most two vertices with odd degree.

**Question 3:** How many different quadruples can one form from the objects  $1, 1, 2, 3, 4, 5, 6, 7, 8, 9$ . Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 4:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: each vertex of side  $A$  is connected with all vertices of side  $B$ . B: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . C: each vertex of side  $B$  is connected to some vertex in side  $A$ .

D: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .

**Question 5:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $m^n$  B:  $m \cdot n$  C:  $n(n-1) \cdots (n-m+1)$  D:  $n^m$

**Question 6:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $\frac{20!}{3!}$  B:  $20 \cdot 19 \cdot 18$  C:  $3^{20}$  D:  $20^3$

**Question 7:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

**Question 8:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to 1 exactly when  $i$  is not connected to  $j$  B: equal to the degree of vertex  $i$  C: equal to 1 exactly when there is a path that connect  $i$  to  $j$ . D: equal to 0 exactly when  $i$  is not connected to  $j$

**Question 9:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!4!}$  B:  $10^4$  C:  $\frac{10!}{6!}$  D:  $6!$

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Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m(n-1) + n(m-1)$  B:  $m+n$  C:  $m \cdot n$  D:  $2(m+n)$

**Question 2:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $2^{10}$  B:  $10!$  C:  $10 \times 10$  D:  $11!$

**Question 3:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A: 30 B:  $3^{10}$  C:  $10^3$  D:  $10 \cdot 9 \cdot 8$

**Question 4:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 5:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: each vertex of side  $A$  is connected with all vertices of side  $B$ . B: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . C: each vertex of side  $B$  is connected to some vertex in side  $A$ . D: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .

**Question 6:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it cannot contain cycles. B: it cannot have more than  $n+1$  edges. C: it must have at least  $n-1$  edges. D: it must have at least  $n$  edges.

**Question 7:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $n^m$  B:  $m^n$  C:  $n(n-1) \cdots (n-m+1)$  D:  $m \cdot n$

**Question 8:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $10^4$  B:  $6!$  C:  $\frac{10!}{6!4!}$  D:  $\frac{10!}{6!}$

**Question 9:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1) \cdots (k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$  B:  $\frac{n(n-1) \cdots (n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $3^{20}$  B:  $20 \cdot 19 \cdot 18$  C:  $20^3$  D:  $\frac{20!}{3!}$

**Question 2:** The binomial coefficient  $\binom{n}{k}$  equals

A: 0 if  $k = 0$ . B:  $\binom{n}{n-k}$ .

**Question 3:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!}$  B:  $10^4$  C:  $\frac{10!}{6!4!}$  D:  $6!$

**Question 4:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 5:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . B: each vertex of side  $B$  is connected to some vertex in side  $A$ . C: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . D: each vertex of side  $A$  is connected with all vertices of side  $B$ .

**Question 6:** In a simple graph with 100 vertices

A: the minimum vertex degree is  $\geq 1$ . B: it is possible that all vertices have different degrees. C: the maximum vertex degree is  $\leq 99$ . D: not all vertex degrees can be odd.

**Question 7:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $3^{11}$  B:  $10!$  C:  $11!$  D:  $9!$

**Question 8:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m(n-1) + n(m-1)$  B:  $m+n$  C:  $m \cdot n$  D:  $2(m+n)$

**Question 9:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $2^n$  B:  $\binom{n}{n/2}$  C:  $3^n$  D:  $2^n + 2^n$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $6!$  B:  $10^4$  C:  $\frac{10!}{6!}$  D:  $\frac{10!}{6!4!}$

**Question 2:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $3^{20}$  B:  $20^3$  C:  $\frac{20!}{3!}$  D:  $20 \cdot 19 \cdot 18$

**Question 3:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 4:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it cannot have more than  $n + 1$  edges. B: it must have at least  $n - 1$  edges. C: it must have at least  $n$  edges. D: it cannot contain cycles.

**Question 5:** In a simple graph with 100 vertices

A: the minimum vertex degree is  $\geq 1$ . B: not all vertex degrees can be odd. C: it is possible that all vertices have different degrees. D: the maximum vertex degree is  $\leq 99$ .

**Question 6:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $3^{10}$  B: 30 C:  $10^3$  D:  $10 \cdot 9 \cdot 8$

**Question 7:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: each vertex of side  $B$  is connected to some vertex in side  $A$ . B: each vertex of side  $A$  is connected with all vertices of side  $B$ . C: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . D: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .

**Question 8:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $9!$  B:  $11!$  C:  $10!$  D:  $3^{11}$

**Question 9:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

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Instructor: Mihalis Kolountzakis

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In a simple graph with 100 vertices

*A:* the minimum vertex degree is  $\geq 1$ . *B:* the maximum vertex degree is  $\leq 99$ . *C:* it is possible that all vertices have different degrees. *D:* not all vertex degrees can be odd.

**Question 2:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

*A:*  $2^n$  *B:*  $2^n + 2^n$  *C:*  $\binom{n}{n/2}$  *D:*  $3^n$

**Question 3:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

*A:*  $\frac{k(k-1)\cdots(k-n+1)}{n\cdot(n-1)\cdots 2\cdot 1}$  *B:*  $\frac{n(n-1)\cdots(n-k+1)}{k\cdot(k-1)\cdots 2\cdot 1}$

**Question 4:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

*A:*  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  *B:*  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 5:** In how many ways can the numbers 0, 1,  $\dots$ , 10 be put in order?

*A:*  $10 \times 10$  *B:*  $2^{10}$  *C:*  $11!$  *D:*  $10!$

**Question 6:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

*A:* side  $B$  has more vertices than side  $A$ . *B:* there is always a perfect matching of the vertices of side  $A$ . *C:* For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ . *D:* side  $A$  has more vertices than side  $B$ .

**Question 7:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

*A:*  $10^3$  *B:* 30 *C:*  $3^{10}$  *D:*  $10 \cdot 9 \cdot 8$

**Question 8:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

*A:*  $6!$  *B:*  $\frac{10!}{6!4!}$  *C:*  $10^4$  *D:*  $\frac{10!}{6!}$

**Question 9:** If  $G$  is a connected simple graph with  $n$  vertices then

*A:* it cannot contain cycles. *B:* it must have at least  $n - 1$  edges. *C:* it must have at least  $n$  edges. *D:* it cannot have more than  $n + 1$  edges.

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Instructor: Mihalis Kolountzakis

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 2:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A: 30 B:  $10 \cdot 9 \cdot 8$  C:  $10^3$  D:  $3^{10}$

**Question 3:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to the degree of vertex  $i$  B: equal to 0 exactly when  $i$  is not connected to  $j$  C: equal to 1 exactly when there is a path that connect  $i$  to  $j$ . D: equal to 1 exactly when  $i$  is not connected to  $j$

**Question 4:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $m \cdot n$  B:  $m^n$  C:  $n^m$  D:  $n(n-1) \cdots (n-m+1)$

**Question 5:** If  $G$  is a simple graph then

A: the number of its vertices with even degree is even. B: it has at least two vertices with odd degree. C: it has at most two vertices with odd degree. D: the number of its vertices with odd degree is not odd.

**Question 6:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $10 \times 10$  B:  $11!$  C:  $10!$  D:  $2^{10}$

**Question 7:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1) \cdots (k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$  B:  $\frac{n(n-1) \cdots (n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$

**Question 8:** In a simple graph with 100 vertices

A: not all vertex degrees can be odd. B: it is possible that all vertices have different degrees. C: the maximum vertex degree is  $\leq 99$ . D: the minimum vertex degree is  $\geq 1$ .

**Question 9:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $6!$  B:  $10^4$  C:  $\frac{10!}{6!}$  D:  $\frac{10!}{6!4!}$

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Instructor: Mihalis Kolountzakis

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$   
 $A$ : the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .  $B$ : the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .  $C$ : each vertex of side  $B$  is connected to some vertex in side  $A$ .  $D$ : each vertex of side  $A$  is connected with all vertices of side  $B$ .

**Question 2:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

$A: 2(m+n)$   $B: m(n-1) + n(m-1)$   $C: m \cdot n$   $D: m+n$

**Question 3:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

$A: m \cdot n$   $B: m^n$   $C: n^m$   $D: n(n-1) \cdots (n-m+1)$

**Question 4:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

$A: \frac{n(n-1) \cdots (n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$   $B: \frac{k(k-1) \cdots (k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$

**Question 5:** If  $G$  is a connected simple graph with  $n$  vertices then

$A$ : it cannot contain cycles.  $B$ : it cannot have more than  $n+1$  edges.  $C$ : it must have at least  $n$  edges.  $D$ : it must have at least  $n-1$  edges.

**Question 6:** The binomial coefficient  $\binom{n}{k}$  equals

$A: 0$  if  $k=0$ .  $B: \binom{n}{n-k}$ .

**Question 7:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

$A: \frac{10!}{6!4!}$   $B: \frac{10!}{6!}$   $C: 10^4$   $D: 6!$

**Question 8:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

$A: 10!$   $B: 3^{11}$   $C: 9!$   $D: 11!$

**Question 9:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

$A: 20^3$   $B: 20 \cdot 19 \cdot 18$   $C: 3^{20}$   $D: \frac{20!}{3!}$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $11!$  B:  $9!$  C:  $3^{11}$  D:  $10!$

**Question 2:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $n(n-1) \cdots (n-m+1)$  B:  $m^n$  C:  $m \cdot n$  D:  $n^m$

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A: it cannot have more than  $n+1$  edges. B: it must have at least  $n-1$  edges. C: it cannot contain cycles. D: it must have at least  $n$  edges.

**Question 4:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . B: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . C: each vertex of side  $B$  is connected to some vertex in side  $A$ . D: each vertex of side  $A$  is connected with all vertices of side  $B$ .

**Question 5:** How many different quadruples can one form from the objects  $1, 1, 2, 3, 4, 5, 6, 7, 8, 9$ . Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 6:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to the degree of vertex  $i$  B: equal to 1 exactly when  $i$  is not connected to  $j$  C: equal to 1 exactly when there is a path that connect  $i$  to  $j$ . D: equal to 0 exactly when  $i$  is not connected to  $j$

**Question 7:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $2^{10}$  B:  $10!$  C:  $11!$  D:  $10 \times 10$

**Question 8:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1) \cdots (k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$  B:  $\frac{n(n-1) \cdots (n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$

**Question 9:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $\binom{n}{n/2}$  B:  $3^n$  C:  $2^n + 2^n$  D:  $2^n$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $10 \cdot 9 \cdot 8$  B:  $10^3$  C:  $3^{10}$  D: 30

**Question 2:** If  $G$  is a simple graph then

A: it has at least two vertices with odd degree. B: the number of its vertices with even degree is even.  
C: it has at most two vertices with odd degree. D: the number of its vertices with odd degree is not odd.

**Question 3:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $3^{11}$  B:  $9!$  C:  $10!$  D:  $11!$

**Question 4:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: side  $B$  has more vertices than side  $A$ . B: there is always a perfect matching of the vertices of side  $A$ .  
C: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ . D: side  $A$  has more vertices than side  $B$ .

**Question 5:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $10^4$  B:  $6!$  C:  $\frac{10!}{6!4!}$  D:  $\frac{10!}{6!}$

**Question 6:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $2^n + 2^n$  B:  $2^n$  C:  $3^n$  D:  $\binom{n}{n/2}$

**Question 7:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 8:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

**Question 9:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it cannot have more than  $n + 1$  edges. B: it must have at least  $n$  edges. C: it must have at least  $n - 1$  edges. D: it cannot contain cycles.

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $20^3$  B:  $3^{20}$  C:  $20 \cdot 19 \cdot 18$  D:  $\frac{20!}{3!}$

**Question 2:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $9!$  B:  $11!$  C:  $10!$  D:  $3^{11}$

**Question 3:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $10!$  B:  $10 \times 10$  C:  $2^{10}$  D:  $11!$

**Question 4:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $2^n$  B:  $3^n$  C:  $2^n + 2^n$  D:  $\binom{n}{n/2}$

**Question 5:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m \cdot n$  B:  $m + n$  C:  $2(m + n)$  D:  $m(n - 1) + n(m - 1)$

**Question 6:** How many different quadruples can one form from the objects  $1, 1, 2, 3, 4, 5, 6, 7, 8, 9$ . Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 7:** The binomial coefficient  $\binom{n}{k}$  equals

A: 0 if  $k = 0$ . B:  $\binom{n}{n-k}$ .

**Question 8:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . B: each vertex of side  $A$  is connected with all vertices of side  $B$ . C: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . D: each vertex of side  $B$  is connected to some vertex in side  $A$ .

**Question 9:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it cannot contain cycles. B: it cannot have more than  $n + 1$  edges. C: it must have at least  $n$  edges. D: it must have at least  $n - 1$  edges.

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Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

**Question 2:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $m^n$  B:  $m \cdot n$  C:  $n(n-1) \cdots (n-m+1)$  D:  $n^m$

**Question 3:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 4:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m \cdot n$  B:  $m + n$  C:  $m(n-1) + n(m-1)$  D:  $2(m+n)$

**Question 5:** If  $G$  is a simple graph then

A: it has at most two vertices with odd degree. B: the number of its vertices with even degree is even.  
C: the number of its vertices with odd degree is not odd. D: it has at least two vertices with odd degree.

**Question 6:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . B: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . C: each vertex of side  $A$  is connected with all vertices of side  $B$ . D: each vertex of side  $B$  is connected to some vertex in side  $A$ .

**Question 7:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $10 \cdot 9 \cdot 8$  B: 30 C:  $3^{10}$  D:  $10^3$

**Question 8:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $11!$  B:  $3^{11}$  C:  $10!$  D:  $9!$

**Question 9:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!}$  B:  $10^4$  C:  $\frac{10!}{6!4!}$  D:  $6!$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $11!$  B:  $3^{11}$  C:  $9!$  D:  $10!$

**Question 2:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . B: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . C: each vertex of side  $B$  is connected to some vertex in side  $A$ . D: each vertex of side  $A$  is connected with all vertices of side  $B$ .

**Question 3:** How many different quadruples can one form from the objects  $1, 1, 2, 3, 4, 5, 6, 7, 8, 9$ . Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 4:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

**Question 5:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $\binom{n}{n/2}$  B:  $2^n$  C:  $2^n + 2^n$  D:  $3^n$

**Question 6:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $3^{20}$  B:  $20^3$  C:  $\frac{20!}{3!}$  D:  $20 \cdot 19 \cdot 18$

**Question 7:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $2(m+n)$  B:  $m+n$  C:  $m \cdot n$  D:  $m(n-1) + n(m-1)$

**Question 8:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it cannot contain cycles. B: it must have at least  $n$  edges. C: it must have at least  $n-1$  edges.

D: it cannot have more than  $n+1$  edges.

**Question 9:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $2^{10}$  B:  $10!$  C:  $10 \times 10$  D:  $11!$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** If  $G$  is a simple graph then

$A$ : it has at least two vertices with odd degree.  $B$ : it has at most two vertices with odd degree.  $C$ : the number of its vertices with even degree is even.  $D$ : the number of its vertices with odd degree is not odd.

**Question 2:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

$A$ :  $10 \times 10$   $B$ :  $10!$   $C$ :  $2^{10}$   $D$ :  $11!$

**Question 3:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

$A$ :  $\frac{n(n-1)\cdots(n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$   $B$ :  $\frac{k(k-1)\cdots(k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$

**Question 4:** The binomial coefficient  $\binom{n}{k}$  equals

$A$ : 0 if  $k = 0$ .  $B$ :  $\binom{n}{n-k}$ .

**Question 5:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

$A$ : 30  $B$ :  $10^3$   $C$ :  $3^{10}$   $D$ :  $10 \cdot 9 \cdot 8$

**Question 6:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

$A$ :  $3^n$   $B$ :  $\binom{n}{n/2}$   $C$ :  $2^n$   $D$ :  $2^n + 2^n$

**Question 7:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

$A$ : equal to 1 exactly when  $i$  is not connected to  $j$   $B$ : equal to 1 exactly when there is a path that connect  $i$  to  $j$ .  $C$ : equal to the degree of vertex  $i$   $D$ : equal to 0 exactly when  $i$  is not connected to  $j$

**Question 8:** If  $G$  is a connected simple graph with  $n$  vertices then

$A$ : it cannot contain cycles.  $B$ : it must have at least  $n$  edges.  $C$ : it cannot have more than  $n + 1$  edges.  $D$ : it must have at least  $n - 1$  edges.

**Question 9:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

$A$ :  $6!$   $B$ :  $\frac{10!}{6!4!}$   $C$ :  $10^4$   $D$ :  $\frac{10!}{6!}$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $10^4$  B:  $\frac{10!}{6!4!}$  C:  $\frac{10!}{6!}$  D:  $6!$

**Question 2:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $10!$  B:  $11!$  C:  $10 \times 10$  D:  $2^{10}$

**Question 3:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $10 \cdot 9 \cdot 8$  B:  $10^3$  C:  $3^{10}$  D: 30

**Question 4:** In a simple graph with 100 vertices

A: it is possible that all vertices have different degrees. B: the maximum vertex degree is  $\leq 99$ . C: not all vertex degrees can be odd. D: the minimum vertex degree is  $\geq 1$ .

**Question 5:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to 1 exactly when  $i$  is not connected to  $j$  B: equal to 0 exactly when  $i$  is not connected to  $j$   
C: equal to 1 exactly when there is a path that connect  $i$  to  $j$ . D: equal to the degree of vertex  $i$

**Question 6:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it must have at least  $n$  edges. B: it cannot contain cycles. C: it must have at least  $n - 1$  edges.  
D: it cannot have more than  $n + 1$  edges.

**Question 7:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

**Question 8:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1)\dots(k-n+1)}{n \cdot (n-1) \dots 2 \cdot 1}$  B:  $\frac{n(n-1)\dots(n-k+1)}{k \cdot (k-1) \dots 2 \cdot 1}$

**Question 9:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $3^{20}$  B:  $20 \cdot 19 \cdot 18$  C:  $\frac{20!}{3!}$  D:  $20^3$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 2:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it must have at least  $n - 1$  edges. B: it must have at least  $n$  edges. C: it cannot contain cycles.

D: it cannot have more than  $n + 1$  edges.

**Question 3:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $10!$  B:  $10 \times 10$  C:  $2^{10}$  D:  $11!$

**Question 4:** If  $G$  is a simple graph then

A: it has at most two vertices with odd degree. B: the number of its vertices with even degree is even.

C: it has at least two vertices with odd degree. D: the number of its vertices with odd degree is not odd.

**Question 5:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $2^n$  B:  $3^n$  C:  $2^n + 2^n$  D:  $\binom{n}{n/2}$

**Question 6:** The binomial coefficient  $\binom{n}{k}$  equals

A: 0 if  $k = 0$ . B:  $\binom{n}{n-k}$ .

**Question 7:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $m^n$  B:  $m \cdot n$  C:  $n^m$  D:  $n(n-1) \cdots (n-m+1)$

**Question 8:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!}$  B:  $\frac{10!}{6!4!}$  C:  $10^4$  D:  $6!$

**Question 9:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: there is always a perfect matching of the vertices of side  $A$ . B: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ . C: side  $A$  has more vertices than side  $B$ . D: side  $B$  has more vertices than side  $A$ .

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Name:

UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $3^n$  B:  $2^n + 2^n$  C:  $\binom{n}{n/2}$  D:  $2^n$

**Question 2:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $3^{11}$  B:  $10!$  C:  $11!$  D:  $9!$

**Question 3:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m(n-1) + n(m-1)$  B:  $m \cdot n$  C:  $2(m+n)$  D:  $m+n$

**Question 4:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $2^{10}$  B:  $10 \times 10$  C:  $11!$  D:  $10!$

**Question 5:** How many different quadruples can one form from the objects  $1, 1, 2, 3, 4, 5, 6, 7, 8, 9$ . Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 6:** In a simple graph with 100 vertices

A: the minimum vertex degree is  $\geq 1$ . B: the maximum vertex degree is  $\leq 99$ . C: it is possible that all vertices have different degrees. D: not all vertex degrees can be odd.

**Question 7:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $n^m$  B:  $m^n$  C:  $n(n-1) \cdots (n-m+1)$  D:  $m \cdot n$

**Question 8:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

**Question 9:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to the degree of vertex  $i$  B: equal to 0 exactly when  $i$  is not connected to  $j$  C: equal to 1 exactly when  $i$  is not connected to  $j$  D: equal to 1 exactly when there is a path that connect  $i$  to  $j$ .

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Instructor: Mihalis Kolountzakis

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $10!$  B:  $11!$  C:  $9!$  D:  $3^{11}$

**Question 2:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $3^n$  B:  $2^n$  C:  $\binom{n}{n/2}$  D:  $2^n + 2^n$

**Question 3:** In a simple graph with 100 vertices

A: it is possible that all vertices have different degrees. B: the minimum vertex degree is  $\geq 1$ . C: not all vertex degrees can be odd. D: the maximum vertex degree is  $\leq 99$ .

**Question 4:** The binomial coefficient  $\binom{n}{k}$  equals

A: 0 if  $k = 0$ . B:  $\binom{n}{n-k}$ .

**Question 5:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $11!$  B:  $10 \times 10$  C:  $2^{10}$  D:  $10!$

**Question 6:** If  $G$  is a simple graph then

A: the number of its vertices with even degree is even. B: the number of its vertices with odd degree is not odd. C: it has at least two vertices with odd degree. D: it has at most two vertices with odd degree.

**Question 7:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1)\dots(k-n+1)}{n \cdot (n-1) \dots 2 \cdot 1}$  B:  $\frac{n(n-1)\dots(n-k+1)}{k \cdot (k-1) \dots 2 \cdot 1}$

**Question 8:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $n^m$  B:  $m \cdot n$  C:  $m^n$  D:  $n(n-1) \dots (n-m+1)$

**Question 9:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m \cdot n$  B:  $2(m+n)$  C:  $m+n$  D:  $m(n-1) + n(m-1)$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m \cdot n$  B:  $m(n-1) + n(m-1)$  C:  $m+n$  D:  $2(m+n)$

**Question 2:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $2^{10}$  B:  $11!$  C:  $10 \times 10$  D:  $10!$

**Question 3:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $\frac{20!}{3!}$  B:  $20 \cdot 19 \cdot 18$  C:  $3^{20}$  D:  $20^3$

**Question 4:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{n(n-1)\dots(n-k+1)}{k \cdot (k-1) \dots 2 \cdot 1}$  B:  $\frac{k(k-1)\dots(k-n+1)}{n \cdot (n-1) \dots 2 \cdot 1}$

**Question 5:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to 1 exactly when  $i$  is not connected to  $j$  B: equal to the degree of vertex  $i$  C: equal to 0 exactly when  $i$  is not connected to  $j$  D: equal to 1 exactly when there is a path that connect  $i$  to  $j$ .

**Question 6:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $3^n$  B:  $\binom{n}{n/2}$  C:  $2^n$  D:  $2^n + 2^n$

**Question 7:** If  $G$  is a simple graph then

A: it has at least two vertices with odd degree. B: the number of its vertices with even degree is even. C: it has at most two vertices with odd degree. D: the number of its vertices with odd degree is not odd.

**Question 8:** How many different quadruples can one form from the objects  $1, 1, 2, 3, 4, 5, 6, 7, 8, 9$ . Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 9:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $10^4$  B:  $\frac{10!}{6!4!}$  C:  $6!$  D:  $\frac{10!}{6!}$

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Instructor: Mihalis Kolountzakis

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 2:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $3^n$  B:  $2^n + 2^n$  C:  $2^n$  D:  $\binom{n}{n/2}$

**Question 3:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

**Question 4:** In a simple graph with 100 vertices

A: it is possible that all vertices have different degrees. B: the minimum vertex degree is  $\geq 1$ . C: the maximum vertex degree is  $\leq 99$ . D: not all vertex degrees can be odd.

**Question 5:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m(n-1) + n(m-1)$  B:  $2(m+n)$  C:  $m+n$  D:  $m \cdot n$

**Question 6:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . B: each vertex of side  $B$  is connected to some vertex in side  $A$ . C: each vertex of side  $A$  is connected with all vertices of side  $B$ . D: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .

**Question 7:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $10^4$  B:  $6!$  C:  $\frac{10!}{6!}$  D:  $\frac{10!}{6!4!}$

**Question 8:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $10 \times 10$  B:  $2^{10}$  C:  $11!$  D:  $10!$

**Question 9:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $3^{10}$  B:  $10 \cdot 9 \cdot 8$  C: 30 D:  $10^3$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $20 \cdot 19 \cdot 18$  B:  $20^3$  C:  $\frac{20!}{3!}$  D:  $3^{20}$

**Question 2:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $3^{10}$  B:  $10 \cdot 9 \cdot 8$  C:  $10^3$  D: 30

**Question 3:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: side  $A$  has more vertices than side  $B$ . B: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ . C: side  $B$  has more vertices than side  $A$ . D: there is always a perfect matching of the vertices of side  $A$ .

**Question 4:** If  $G$  is a simple graph then

A: it has at most two vertices with odd degree. B: it has at least two vertices with odd degree. C: the number of its vertices with even degree is even. D: the number of its vertices with odd degree is not odd.

**Question 5:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!4!}$  B:  $\frac{10!}{6!}$  C:  $6!$  D:  $10^4$

**Question 6:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 7:** In a simple graph with 100 vertices

A: the maximum vertex degree is  $\leq 99$ . B: the minimum vertex degree is  $\geq 1$ . C: not all vertex degrees can be odd. D: it is possible that all vertices have different degrees.

**Question 8:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1)\dots(k-n+1)}{n \cdot (n-1) \dots 2 \cdot 1}$  B:  $\frac{n(n-1)\dots(n-k+1)}{k \cdot (k-1) \dots 2 \cdot 1}$

**Question 9:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $2^{10}$  B:  $10!$  C:  $11!$  D:  $10 \times 10$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 2:** In a simple graph with 100 vertices

A: the maximum vertex degree is  $\leq 99$ . B: the minimum vertex degree is  $\geq 1$ . C: not all vertex degrees can be odd. D: it is possible that all vertices have different degrees.

**Question 3:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

**Question 4:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $m^n$  B:  $m \cdot n$  C:  $n(n-1) \cdots (n-m+1)$  D:  $n^m$

**Question 5:** How many circular orderings of the numbers 0, 1, ..., 10 are there? (Two circular orderings which differ only by a rotation are not considered different.)

A: 9! B: 10! C: 11! D:  $3^{11}$

**Question 6:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . B: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . C: each vertex of side  $B$  is connected to some vertex in side  $A$ . D: each vertex of side  $A$  is connected with all vertices of side  $B$ .

**Question 7:** In how many ways can the numbers 0, 1, ..., 10 be put in order?

A:  $2^{10}$  B: 10! C:  $10 \times 10$  D: 11!

**Question 8:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m(n-1) + n(m-1)$  B:  $m \cdot n$  C:  $2(m+n)$  D:  $m+n$

**Question 9:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $20^3$  B:  $\frac{20!}{3!}$  C:  $3^{20}$  D:  $20 \cdot 19 \cdot 18$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $6!$  B:  $\frac{10!}{6!}$  C:  $\frac{10!}{6!4!}$  D:  $10^4$

**Question 2:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $\frac{20!}{3!}$  B:  $20 \cdot 19 \cdot 18$  C:  $20^3$  D:  $3^{20}$

**Question 3:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it must have at least  $n - 1$  edges. B: it cannot have more than  $n + 1$  edges. C: it cannot contain cycles. D: it must have at least  $n$  edges.

**Question 4:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: side  $B$  has more vertices than side  $A$ . B: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ . C: side  $A$  has more vertices than side  $B$ . D: there is always a perfect matching of the vertices of side  $A$ .

**Question 5:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A: 30 B:  $3^{10}$  C:  $10 \cdot 9 \cdot 8$  D:  $10^3$

**Question 6:** In a simple graph with 100 vertices

A: the minimum vertex degree is  $\geq 1$ . B: not all vertex degrees can be odd. C: the maximum vertex degree is  $\leq 99$ . D: it is possible that all vertices have different degrees.

**Question 7:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 8:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1)\dots(k-n+1)}{n \cdot (n-1) \dots 2 \cdot 1}$  B:  $\frac{n(n-1)\dots(n-k+1)}{k \cdot (k-1) \dots 2 \cdot 1}$

**Question 9:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $10!$  B:  $10 \times 10$  C:  $2^{10}$  D:  $11!$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $6!$  B:  $\frac{10!}{6!}$  C:  $10^4$  D:  $\frac{10!}{6!4!}$

**Question 2:** In a simple graph with 100 vertices

A: it is possible that all vertices have different degrees. B: not all vertex degrees can be odd. C: the minimum vertex degree is  $\geq 1$ . D: the maximum vertex degree is  $\leq 99$ .

**Question 3:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $m^n$  B:  $n(n-1) \cdots (n-m+1)$  C:  $n^m$  D:  $m \cdot n$

**Question 4:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

**Question 5:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 6:** In how many ways can the numbers 0, 1,  $\dots$ , 10 be put in order?

A:  $11!$  B:  $10!$  C:  $10 \times 10$  D:  $2^{10}$

**Question 7:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $10^3$  B: 30 C:  $3^{10}$  D:  $10 \cdot 9 \cdot 8$

**Question 8:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it must have at least  $n - 1$  edges. B: it cannot have more than  $n + 1$  edges. C: it cannot contain cycles. D: it must have at least  $n$  edges.

**Question 9:** If  $G$  is a simple graph then

A: the number of its vertices with odd degree is not odd. B: the number of its vertices with even degree is even. C: it has at least two vertices with odd degree. D: it has at most two vertices with odd degree.

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** The binomial coefficient  $\binom{n}{k}$  equals

A: 0 if  $k = 0$ . B:  $\binom{n}{n-k}$ .

**Question 2:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it cannot contain cycles. B: it must have at least  $n - 1$  edges. C: it must have at least  $n$  edges.  
D: it cannot have more than  $n + 1$  edges.

**Question 3:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: side  $A$  has more vertices than side  $B$ . B: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ . C: there is always a perfect matching of the vertices of side  $A$ . D: side  $B$  has more vertices than side  $A$ .

**Question 4:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $2^n + 2^n$  B:  $\binom{n}{n/2}$  C:  $3^n$  D:  $2^n$

**Question 5:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $3^{10}$  B:  $10^3$  C: 30 D:  $10 \cdot 9 \cdot 8$

**Question 6:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!4!}$  B:  $\frac{10!}{6!}$  C:  $10^4$  D:  $6!$

**Question 7:** In a simple graph with 100 vertices

A: the maximum vertex degree is  $\leq 99$ . B: the minimum vertex degree is  $\geq 1$ . C: it is possible that all vertices have different degrees. D: not all vertex degrees can be odd.

**Question 8:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 9:** In how many ways can the numbers 0, 1,  $\dots$ , 10 be put in order?

A:  $11!$  B:  $10 \times 10$  C:  $2^{10}$  D:  $10!$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$   
 $A$ : each vertex of side  $A$  is connected with all vertices of side  $B$ .  $B$ : the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .  $C$ : the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .  $D$ : each vertex of side  $B$  is connected to some vertex in side  $A$ .

**Question 2:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

$$A: \frac{k(k-1)\cdots(k-n+1)}{n\cdot(n-1)\cdots 2\cdot 1} \quad B: \frac{n(n-1)\cdots(n-k+1)}{k\cdot(k-1)\cdots 2\cdot 1}$$

**Question 3:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

$$A: 10! \quad B: 10 \times 10 \quad C: 2^{10} \quad D: 11!$$

**Question 4:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

$A$ : there is always a perfect matching of the vertices of side  $A$ .  $B$ : side  $B$  has more vertices than side  $A$ .  
 $C$ : For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ .  $D$ : side  $A$  has more vertices than side  $B$ .

**Question 5:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

$$A: 3^{10} \quad B: 10 \cdot 9 \cdot 8 \quad C: 30 \quad D: 10^3$$

**Question 6:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

$$A: 3^{11} \quad B: 10! \quad C: 9! \quad D: 11!$$

**Question 7:** The binomial coefficient  $\binom{n}{k}$  equals

$$A: \binom{n}{n-k}. \quad B: 0 \text{ if } k = 0.$$

**Question 8:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

$$A: \binom{n}{n/2} \quad B: 3^n \quad C: 2^n \quad D: 2^n + 2^n$$

**Question 9:** If  $G$  is a simple graph then

$A$ : it has at least two vertices with odd degree.  $B$ : the number of its vertices with even degree is even.  
 $C$ : it has at most two vertices with odd degree.  $D$ : the number of its vertices with odd degree is not odd.

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In a simple graph with 100 vertices

*A:* not all vertex degrees can be odd. *B:* the minimum vertex degree is  $\geq 1$ . *C:* the maximum vertex degree is  $\leq 99$ . *D:* it is possible that all vertices have different degrees.

**Question 2:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

*A:*  $\frac{n(n-1)\cdots(n-k+1)}{k\cdot(k-1)\cdots 2\cdot 1}$  *B:*  $\frac{k(k-1)\cdots(k-n+1)}{n\cdot(n-1)\cdots 2\cdot 1}$

**Question 3:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

*A:*  $20 \cdot 19 \cdot 18$  *B:*  $3^{20}$  *C:*  $20^3$  *D:*  $\frac{20!}{3!}$

**Question 4:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

*A:*  $10 \cdot 9 \cdot 8$  *B:* 30 *C:*  $3^{10}$  *D:*  $10^3$

**Question 5:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

*A:* there is always a perfect matching of the vertices of side  $A$ . *B:* side  $B$  has more vertices than side  $A$ . *C:* For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ . *D:* side  $A$  has more vertices than side  $B$ .

**Question 6:** The binomial coefficient  $\binom{n}{k}$  equals

*A:*  $\binom{n}{n-k}$ . *B:* 0 if  $k = 0$ .

**Question 7:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

*A:*  $2(m+n)$  *B:*  $m \cdot n$  *C:*  $m+n$  *D:*  $m(n-1) + n(m-1)$

**Question 8:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

*A:*  $\frac{10!}{6!}$  *B:*  $6!$  *C:*  $\frac{10!}{6!4!}$  *D:*  $10^4$

**Question 9:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

*A:*  $10 \times 10$  *B:*  $2^{10}$  *C:*  $10!$  *D:*  $11!$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!4!}$  B:  $6!$  C:  $\frac{10!}{6!}$  D:  $10^4$

**Question 2:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to 0 exactly when  $i$  is not connected to  $j$  B: equal to 1 exactly when there is a path that connect  $i$  to  $j$ . C: equal to 1 exactly when  $i$  is not connected to  $j$  D: equal to the degree of vertex  $i$

**Question 3:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 4:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $2^n$  B:  $\binom{n}{n/2}$  C:  $3^n$  D:  $2^n + 2^n$

**Question 5:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it must have at least  $n - 1$  edges. B: it must have at least  $n$  edges. C: it cannot have more than  $n + 1$  edges. D: it cannot contain cycles.

**Question 6:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1)\dots(k-n+1)}{n \cdot (n-1) \dots 2 \cdot 1}$  B:  $\frac{n(n-1)\dots(n-k+1)}{k \cdot (k-1) \dots 2 \cdot 1}$

**Question 7:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $10^3$  B:  $3^{10}$  C: 30 D:  $10 \cdot 9 \cdot 8$

**Question 8:** In a simple graph with 100 vertices

A: the minimum vertex degree is  $\geq 1$ . B: not all vertex degrees can be odd. C: it is possible that all vertices have different degrees. D: the maximum vertex degree is  $\leq 99$ .

**Question 9:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $2^{10}$  B:  $11!$  C:  $10 \times 10$  D:  $10!$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In a simple graph with 100 vertices

*A:* the maximum vertex degree is  $\leq 99$ . *B:* not all vertex degrees can be odd. *C:* the minimum vertex degree is  $\geq 1$ . *D:* it is possible that all vertices have different degrees.

**Question 2:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

*A:* equal to the degree of vertex  $i$  *B:* equal to 1 exactly when there is a path that connect  $i$  to  $j$ . *C:* equal to 1 exactly when  $i$  is not connected to  $j$  *D:* equal to 0 exactly when  $i$  is not connected to  $j$

**Question 3:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

*A:*  $20 \cdot 19 \cdot 18$  *B:*  $3^{20}$  *C:*  $20^3$  *D:*  $\frac{20!}{3!}$

**Question 4:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

*A:* each vertex of side  $A$  is connected with all vertices of side  $B$ . *B:* the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . *C:* each vertex of side  $B$  is connected to some vertex in side  $A$ . *D:* the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .

**Question 5:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

*A:*  $3^n$  *B:*  $2^n + 2^n$  *C:*  $2^n$  *D:*  $\binom{n}{n/2}$

**Question 6:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

*A:*  $10 \cdot 9 \cdot 8 \cdot 7$  *B:*  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 7:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

*A:*  $9!$  *B:*  $3^{11}$  *C:*  $10!$  *D:*  $11!$

**Question 8:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

*A:*  $10 \times 10$  *B:*  $10!$  *C:*  $11!$  *D:*  $2^{10}$

**Question 9:** The binomial coefficient  $\binom{n}{k}$  equals

*A:*  $\binom{n}{n-k}$ . *B:* 0 if  $k = 0$ .

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Instructor: Mihalis Kolountzakis

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $11!$  B:  $10 \times 10$  C:  $10!$  D:  $2^{10}$

**Question 2:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: each vertex of side  $A$  is connected with all vertices of side  $B$ . B: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . C: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . D: each vertex of side  $B$  is connected to some vertex in side  $A$ .

**Question 3:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

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**Question 4:** The binomial coefficient  $\binom{n}{k}$  equals

A: 0 if  $k = 0$ . B:  $\binom{n}{n-k}$ .

**Question 5:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!}$  B:  $6!$  C:  $\frac{10!}{6!4!}$  D:  $10^4$

**Question 6:** In a simple graph with 100 vertices

A: the maximum vertex degree is  $\leq 99$ . B: not all vertex degrees can be odd. C: it is possible that all vertices have different degrees. D: the minimum vertex degree is  $\geq 1$ .

**Question 7:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $9!$  B:  $3^{11}$  C:  $11!$  D:  $10!$

**Question 2:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!}$  B:  $6!$  C:  $10^4$  D:  $\frac{10!}{6!4!}$

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A:  $2^n + 2^n$  B:  $\binom{n}{n/2}$  C:  $3^n$  D:  $2^n$

**Question 4:** In a simple graph with 100 vertices

A: it is possible that all vertices have different degrees. B: the minimum vertex degree is  $\geq 1$ . C: not all vertex degrees can be odd. D: the maximum vertex degree is  $\leq 99$ .

**Question 5:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1)\cdots(k-n+1)}{n\cdot(n-1)\cdots 2\cdot 1}$  B:  $\frac{n(n-1)\cdots(n-k+1)}{k\cdot(k-1)\cdots 2\cdot 1}$

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**Question 9:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

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Name:

UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $3^n$  B:  $2^n$  C:  $\binom{n}{n/2}$  D:  $2^n + 2^n$

**Question 2:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!4!}$  B:  $\frac{10!}{6!}$  C:  $6!$  D:  $10^4$

**Question 3:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $n^m$  B:  $m^n$  C:  $m \cdot n$  D:  $n(n-1) \cdots (n-m+1)$

**Question 4:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{n(n-1) \cdots (n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$  B:  $\frac{k(k-1) \cdots (k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$

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A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

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A:  $m \cdot n$  B:  $2(m+n)$  C:  $m+n$  D:  $m(n-1) + n(m-1)$

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Instructor: Mihalis Kolountzakis

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

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A: it cannot have more than  $n + 1$  edges. B: it must have at least  $n - 1$  edges. C: it cannot contain cycles. D: it must have at least  $n$  edges.

**Question 6:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $10 \cdot 9 \cdot 8$  B: 30 C:  $3^{10}$  D:  $10^3$

**Question 7:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: there is always a perfect matching of the vertices of side  $A$ . B: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ . C: side  $B$  has more vertices than side  $A$ . D: side  $A$  has more vertices than side  $B$ .

**Question 8:** In a simple graph with 100 vertices

A: not all vertex degrees can be odd. B: the maximum vertex degree is  $\leq 99$ . C: it is possible that all vertices have different degrees. D: the minimum vertex degree is  $\geq 1$ .

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A:  $10!$  B:  $2^{10}$  C:  $10 \times 10$  D:  $11!$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

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**Question 3:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . B: each vertex of side  $B$  is connected to some vertex in side  $A$ . C: each vertex of side  $A$  is connected with all vertices of side  $B$ . D: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .

**Question 4:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

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**Question 7:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m \cdot n$  B:  $m + n$  C:  $2(m + n)$  D:  $m(n - 1) + n(m - 1)$

**Question 8:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $3^{11}$  B:  $10!$  C:  $9!$  D:  $11!$

**Question 9:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $2^n + 2^n$  B:  $2^n$  C:  $3^n$  D:  $\binom{n}{n/2}$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $10!$  B:  $11!$  C:  $2^{10}$  D:  $10 \times 10$

**Question 2:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $3^{11}$  B:  $9!$  C:  $10!$  D:  $11!$

**Question 3:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $m \cdot n$  B:  $n(n-1) \cdots (n-m+1)$  C:  $m^n$  D:  $n^m$

**Question 4:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

**Question 5:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m+n$  B:  $m(n-1) + n(m-1)$  C:  $2(m+n)$  D:  $m \cdot n$

**Question 6:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 7:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $3^{20}$  B:  $\frac{20!}{3!}$  C:  $20 \cdot 19 \cdot 18$  D:  $20^3$

**Question 8:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: each vertex of side  $B$  is connected to some vertex in side  $A$ . B: each vertex of side  $A$  is connected with all vertices of side  $B$ . C: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . D: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .

**Question 9:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to the degree of vertex  $i$  B: equal to 1 exactly when there is a path that connect  $i$  to  $j$ . C: equal to 0 exactly when  $i$  is not connected to  $j$  D: equal to 1 exactly when  $i$  is not connected to  $j$

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Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $10^4$  B:  $\frac{10!}{6!}$  C:  $6!$  D:  $\frac{10!}{6!4!}$

**Question 2:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $9!$  B:  $3^{11}$  C:  $11!$  D:  $10!$

**Question 3:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1)\cdots(k-n+1)}{n\cdot(n-1)\cdots 2\cdot 1}$  B:  $\frac{n(n-1)\cdots(n-k+1)}{k\cdot(k-1)\cdots 2\cdot 1}$

**Question 4:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $\frac{20!}{3!}$  B:  $20^3$  C:  $3^{20}$  D:  $20 \cdot 19 \cdot 18$

**Question 5:** In a simple graph with 100 vertices

A: the minimum vertex degree is  $\geq 1$ . B: not all vertex degrees can be odd. C: it is possible that all vertices have different degrees. D: the maximum vertex degree is  $\leq 99$ .

**Question 6:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $m^n$  B:  $m \cdot n$  C:  $n(n-1)\cdots(n-m+1)$  D:  $n^m$

**Question 7:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ . B: side  $B$  has more vertices than side  $A$ . C: there is always a perfect matching of the vertices of side  $A$ . D: side  $A$  has more vertices than side  $B$ .

**Question 8:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to the degree of vertex  $i$  B: equal to 1 exactly when there is a path that connect  $i$  to  $j$ . C: equal to 1 exactly when  $i$  is not connected to  $j$  D: equal to 0 exactly when  $i$  is not connected to  $j$

**Question 9:** The binomial coefficient  $\binom{n}{k}$  equals

A: 0 if  $k = 0$ . B:  $\binom{n}{n-k}$ .

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $10!$  B:  $11!$  C:  $9!$  D:  $3^{11}$

**Question 2:** How many different quadruples can one form from the objects  $1, 1, 2, 3, 4, 5, 6, 7, 8, 9$ . Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 3:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $n^m$  B:  $m \cdot n$  C:  $m^n$  D:  $n(n-1) \cdots (n-m+1)$

**Question 4:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m(n-1) + n(m-1)$  B:  $2(m+n)$  C:  $m+n$  D:  $m \cdot n$

**Question 5:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

A: there is always a perfect matching of the vertices of side  $A$ . B: side  $B$  has more vertices than side  $A$ . C: side  $A$  has more vertices than side  $B$ . D: For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ .

**Question 6:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $\binom{n}{n/2}$  B:  $2^n$  C:  $3^n$  D:  $2^n + 2^n$

**Question 7:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!}$  B:  $6!$  C:  $10^4$  D:  $\frac{10!}{6!4!}$

**Question 8:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to 1 exactly when there is a path that connect  $i$  to  $j$ . B: equal to 1 exactly when  $i$  is not connected to  $j$  C: equal to the degree of vertex  $i$  D: equal to 0 exactly when  $i$  is not connected to  $j$

**Question 9:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1) \cdots (k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$  B:  $\frac{n(n-1) \cdots (n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

**Question 2:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $2(m+n)$  B:  $m(n-1) + n(m-1)$  C:  $m+n$  D:  $m \cdot n$

**Question 3:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $2^{10}$  B:  $10 \times 10$  C:  $10!$  D:  $11!$

**Question 4:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!4!}$  B:  $10^4$  C:  $\frac{10!}{6!}$  D:  $6!$

**Question 5:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A: 30 B:  $10 \cdot 9 \cdot 8$  C:  $10^3$  D:  $3^{10}$

**Question 6:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 7:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $20 \cdot 19 \cdot 18$  B:  $20^3$  C:  $\frac{20!}{3!}$  D:  $3^{20}$

**Question 8:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: each vertex of side  $A$  is connected with all vertices of side  $B$ . B: each vertex of side  $B$  is connected to some vertex in side  $A$ . C: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ . D: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .

**Question 9:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to 0 exactly when  $i$  is not connected to  $j$  B: equal to 1 exactly when there is a path that connect  $i$  to  $j$ . C: equal to the degree of vertex  $i$  D: equal to 1 exactly when  $i$  is not connected to  $j$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

A:  $m^n$  B:  $n^m$  C:  $n(n-1) \cdots (n-m+1)$  D:  $m \cdot n$

**Question 2:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $2^{10}$  B:  $10 \times 10$  C:  $10!$  D:  $11!$

**Question 3:** In a simple graph with 100 vertices

A: the maximum vertex degree is  $\leq 99$ . B: the minimum vertex degree is  $\geq 1$ . C: it is possible that all vertices have different degrees. D: not all vertex degrees can be odd.

**Question 4:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

A:  $2^n$  B:  $\binom{n}{n/2}$  C:  $3^n$  D:  $2^n + 2^n$

**Question 5:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $6!$  B:  $\frac{10!}{6!}$  C:  $10^4$  D:  $\frac{10!}{6!4!}$

**Question 6:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $10 \cdot 9 \cdot 8 \cdot 7$  B:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$

**Question 7:** If  $G$  is a simple graph then

A: the number of its vertices with odd degree is not odd. B: it has at least two vertices with odd degree. C: the number of its vertices with even degree is even. D: it has at most two vertices with odd degree.

**Question 8:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

A:  $\frac{k(k-1) \cdots (k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$  B:  $\frac{n(n-1) \cdots (n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$

**Question 9:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it cannot have more than  $n+1$  edges. B: it cannot contain cycles. C: it must have at least  $n$  edges. D: it must have at least  $n-1$  edges.

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In a simple graph with 100 vertices

*A:* the maximum vertex degree is  $\leq 99$ . *B:* the minimum vertex degree is  $\geq 1$ . *C:* not all vertex degrees can be odd. *D:* it is possible that all vertices have different degrees.

**Question 2:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

*A:*  $\binom{n}{n/2}$  *B:*  $2^n + 2^n$  *C:*  $3^n$  *D:*  $2^n$

**Question 3:** If  $G$  is a simple graph then

*A:* it has at most two vertices with odd degree. *B:* the number of its vertices with even degree is even. *C:* it has at least two vertices with odd degree. *D:* the number of its vertices with odd degree is not odd.

**Question 4:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

*A:*  $11!$  *B:*  $10!$  *C:*  $2^{10}$  *D:*  $10 \times 10$

**Question 5:** How many different quadruples can one form from the objects  $1, 1, 2, 3, 4, 5, 6, 7, 8, 9$ . Two quadruples differing only in order are not considered different.

*A:*  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  *B:*  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 6:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

*A:*  $\frac{k(k-1)\cdots(k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$  *B:*  $\frac{n(n-1)\cdots(n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$

**Question 7:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

*A:*  $10^4$  *B:*  $6!$  *C:*  $\frac{10!}{6!}$  *D:*  $\frac{10!}{6!4!}$

**Question 8:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

*A:* each vertex of side  $A$  is connected with all vertices of side  $B$ . *B:* the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . *C:* each vertex of side  $B$  is connected to some vertex in side  $A$ . *D:* the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .

**Question 9:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

*A:*  $n^m$  *B:*  $m \cdot n$  *C:*  $m^n$  *D:*  $n(n-1) \cdots (n-m+1)$

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

A:  $11!$  B:  $10 \times 10$  C:  $2^{10}$  D:  $10!$

**Question 2:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!}$  B:  $\frac{10!}{6!4!}$  C:  $10^4$  D:  $6!$

**Question 3:** In how many ways can we select, from a set of 20 people, a committee of 3 different persons with a chair, secretary and member?

A:  $\frac{20!}{3!}$  B:  $20 \cdot 19 \cdot 18$  C:  $20^3$  D:  $3^{20}$

**Question 4:** If  $G$  is a connected simple graph with  $n$  vertices then

A: it must have at least  $n$  edges. B: it cannot have more than  $n + 1$  edges. C: it cannot contain cycles. D: it must have at least  $n - 1$  edges.

**Question 5:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

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A:  $\frac{n(n-1)\dots(n-k+1)}{k \cdot (k-1) \dots 2 \cdot 1}$  B:  $\frac{k(k-1)\dots(k-n+1)}{n \cdot (n-1) \dots 2 \cdot 1}$

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A: equal to 1 exactly when  $i$  is not connected to  $j$  B: equal to 0 exactly when  $i$  is not connected to  $j$   
C: equal to the degree of vertex  $i$  D: equal to 1 exactly when there is a path that connect  $i$  to  $j$ .

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** How many different quadruples can one form from the objects 1, 1, 2, 3, 4, 5, 6, 7, 8, 9. Two quadruples differing only in order are not considered different.

A:  $\binom{8}{4} + \binom{8}{3} + \binom{8}{2}$  B:  $10 \cdot 9 \cdot 8 \cdot 7$

**Question 2:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

A:  $3^{11}$  B:  $10!$  C:  $9!$  D:  $11!$

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A:  $\frac{20!}{3!}$  B:  $3^{20}$  C:  $20^3$  D:  $20 \cdot 19 \cdot 18$

**Question 4:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

A:  $3^{10}$  B:  $10 \cdot 9 \cdot 8$  C:  $10^3$  D: 30

**Question 5:** The binomial coefficient  $\binom{n}{k}$  equals

A:  $\binom{n}{n-k}$ . B: 0 if  $k = 0$ .

**Question 6:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

A: equal to the degree of vertex  $i$  B: equal to 0 exactly when  $i$  is not connected to  $j$  C: equal to 1 exactly when  $i$  is not connected to  $j$  D: equal to 1 exactly when there is a path that connect  $i$  to  $j$ .

**Question 7:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

A:  $\frac{10!}{6!4!}$  B:  $10^4$  C:  $6!$  D:  $\frac{10!}{6!}$

**Question 8:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

A: the number of vertices of side  $A$  is at least the number of vertices of side  $B$ . B: each vertex of side  $A$  is connected with all vertices of side  $B$ . C: each vertex of side  $B$  is connected to some vertex in side  $A$ . D: the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .

**Question 9:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

A:  $m + n$  B:  $2(m + n)$  C:  $m \cdot n$  D:  $m(n - 1) + n(m - 1)$

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The examination lasts 2 hours and all books are closed. Return only this paper with your answers. Record the serial number of your paper and your answers on a piece of paper and keep it. Wrong answers reduce your score. Not answering a question counts as 0. There is precisely one correct answer per question.

Instructor: Mihalis Kolountzakis

Iraklio, 7 February 2004

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

$A$ : equal to the degree of vertex  $i$     $B$ : equal to 1 exactly when  $i$  is not connected to  $j$     $C$ : equal to 0 exactly when  $i$  is not connected to  $j$     $D$ : equal to 1 exactly when there is a path that connect  $i$  to  $j$ .

**Question 2:** If we have 10 distinct objects and we can color each of them red, green or blue, how many different colorings are possible?

$A$ : 30    $B$ :  $3^{10}$     $C$ :  $10 \cdot 9 \cdot 8$     $D$ :  $10^3$

**Question 3:** In how many ways can the numbers  $0, 1, \dots, 10$  be put in order?

$A$ :  $2^{10}$     $B$ :  $10 \times 10$     $C$ :  $10!$     $D$ :  $11!$

**Question 4:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

$A$ :  $n^m$     $B$ :  $n(n-1) \cdots (n-m+1)$     $C$ :  $m^n$     $D$ :  $m \cdot n$

**Question 5:** The number of edges of the complete bipartite graph  $K_{mn}$ , with vertex sets  $A = \{a_1, \dots, a_m\}$  and  $B = \{b_1, \dots, b_n\}$  is

$A$ :  $m \cdot n$     $B$ :  $m + n$     $C$ :  $m(n-1) + n(m-1)$     $D$ :  $2(m+n)$

**Question 6:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

$A$ :  $\frac{10!}{6!}$     $B$ :  $\frac{10!}{6!4!}$     $C$ :  $10^4$     $D$ :  $6!$

**Question 7:** The binomial coefficient  $\binom{n}{k}$  equals

$A$ :  $\binom{n}{n-k}$ .    $B$ : 0 if  $k = 0$ .

**Question 8:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

$A$ :  $\frac{n(n-1) \cdots (n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$     $B$ :  $\frac{k(k-1) \cdots (k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$

**Question 9:** A bipartite graph  $G$  with vertex sets  $A$  and  $B$  is  $r$ -regular. That is all its vertices have the same degree  $r$ . Then

$A$ : there is always a perfect matching of the vertices of side  $A$ .    $B$ : side  $B$  has more vertices than side  $A$ .  
 $C$ : For every subset  $J \subseteq A$  the set of all its neighbors has more elements than  $J$ .    $D$ : side  $A$  has more vertices than side  $B$ .

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UNIVERSITY OF CRETE – DEPARTMENT OF MATHEMATICS – DISCRETE MATHEMATICS I

Final examination

**Question 1:** If  $G$  is a connected simple graph with  $n$  vertices then

$A$ : it cannot have more than  $n + 1$  edges.  $B$ : it must have at least  $n - 1$  edges.  $C$ : it must have at least  $n$  edges.  $D$ : it cannot contain cycles.

**Question 2:** If  $A$  is the adjacency matrix of the simple graph  $G$  with vertex set  $V = \{1, 2, \dots, n\}$ , then the entry  $A_{i,j}$ , with  $i, j \in V$  is

$A$ : equal to 1 exactly when  $i$  is not connected to  $j$   $B$ : equal to the degree of vertex  $i$   $C$ : equal to 0 exactly when  $i$  is not connected to  $j$   $D$ : equal to 1 exactly when there is a path that connect  $i$  to  $j$ .

**Question 3:** In a bipartite graph with vertex sets  $A$  and  $B$  which has a perfect matching of side  $A$

$A$ : each vertex of side  $B$  is connected to some vertex in side  $A$ .  $B$ : the number of vertices of side  $A$  is at least the number of vertices of side  $B$ .  $C$ : each vertex of side  $A$  is connected with all vertices of side  $B$ .  $D$ : the number of vertices of side  $B$  is at least the number of vertices of side  $A$ .

**Question 4:** How many circular orderings of the numbers  $0, 1, \dots, 10$  are there? (Two circular orderings which differ only by a rotation are not considered different.)

$A$ :  $3^{11}$   $B$ :  $9!$   $C$ :  $11!$   $D$ :  $10!$

**Question 5:** In how many ways can we choose 4 numbers from the set  $\{1, \dots, 10\}$  if the order in which we choose them matters?

$A$ :  $\frac{10!}{6!4!}$   $B$ :  $10^4$   $C$ :  $6!$   $D$ :  $\frac{10!}{6!}$

**Question 6:** The binomial coefficient  $\binom{n}{k}$  equals

$A$ : 0 if  $k = 0$ .  $B$ :  $\binom{n}{n-k}$ .

**Question 7:** How many different functions are there from the set  $\{1, \dots, m\}$  to the set  $\{1, \dots, n\}$ ?

$A$ :  $n^m$   $B$ :  $m \cdot n$   $C$ :  $m^n$   $D$ :  $n(n-1) \cdots (n-m+1)$

**Question 8:** In how many ways can we choose  $n$  objects from  $k$  different objects, if the order of choice does not matter?

$A$ :  $\frac{k(k-1) \cdots (k-n+1)}{n \cdot (n-1) \cdots 2 \cdot 1}$   $B$ :  $\frac{n(n-1) \cdots (n-k+1)}{k \cdot (k-1) \cdots 2 \cdot 1}$

**Question 9:** In how many ways can we select two disjoint subsets  $A$  and  $B$  of  $\{1, 2, \dots, n\}$ ? (The internal order in  $A$  and  $B$  is irrelevant, but it matters which set is  $A$  and which is  $B$ .)

$A$ :  $3^n$   $B$ :  $2^n$   $C$ :  $2^n + 2^n$   $D$ :  $\binom{n}{n/2}$

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