Υψηλότερης τάξης ροπές του τυχαίου περίπατου

$S_n = X_1 + X_2 + \cdots + X_n$, $X_j = \pm 1$ με ίση πιθ.

ανεξάρτητα

$\mathbb{E}S_n = 0$, $\mathbb{E}S_n^2 = \sigma^2(S_n) = n$

$\mathbb{E}S_n^3 = \mathbb{E}\left[(X_1 + X_2 + \cdots + X_n)^3\right] = \mathbb{E}\left[(X_1 + \cdots + X_n)(X_1 + \cdots + X_n)(X_1 + \cdots + X_n)\right]$

$= \mathbb{E}\left[\sum_{i,j,k=1}^{n} X_i X_j X_k\right] = \sum_{i,j,k=1}^{n} \mathbb{E}(X_i X_j X_k) = 0$

1) i, j, k όλα διαφορετικά π.χ. $X_1 X_2 X_3 \longrightarrow \mathbb{E}(X_i X_j X_k) = \underbrace{\mathbb{E}X_i}_{=0} \underbrace{\mathbb{E}X_j}_{=0} \underbrace{\mathbb{E}X_k}_{=0} = 0$

2) $i = j = k$ π.χ. $X_1 X_1 X_1 = X_1^3 \longrightarrow \mathbb{E}(X_i^3) = \mathbb{E}X_i = 0$

3) $i = j \neq k$ π.χ. $X_1 X_1 X_2 = X_1^2 X_2 \rightarrow \mathbb{E}(X_i^2 X_j) = \mathbb{E}(X_i^2) \cdot \overset{=0}{\mathbb{E}X_j} = 0$

$\mathbb{E}S_n^4$?

$$\mathbb{E} S_n^4 = \mathbb{E}\left((X_1+X_2+\dots+X_n)(X_1+\dots+X_n)(X_1+\dots+X_n)(X_1+\dots+X_n)\right)$$

$$= \sum_{i,j,k,l=1}^{n} \mathbb{E}(X_i X_j X_k X_l) = \underset{\text{κατ. 1}}{n} + \underset{\text{κατ. 4}}{3n(n-1)} = \underline{\underline{3n^2-2n}}$$

1) $i=j=k=l$ $\quad \mathbb{E}(\underline{X_i^4}) = 1$ πόσα; n

2) $i=j=k\neq l$ $\quad \mathbb{E}(X_i^3 X_j) = \mathbb{E}(X_i^3)\mathbb{E}(X_j) = 0$

3) $i=j,\ k\neq i,\ l\neq i,\ k\neq l$ $\quad \mathbb{E}(X_i^2 X_k X_l) = \mathbb{E}(X_i^2)\,\mathbb{E}X_k\,\mathbb{E}X_l = 0$

$\bullet\ \bullet\ *\ \wedge$

4) $i=j,\ k=l,\ i\neq k$ $\quad \mathbb{E}(\underline{X_i^2 X_k^2}) = \underbrace{\mathbb{E}(X_i^2)}_{=1}\underbrace{\mathbb{E}(X_k^2)}_{=1} = 1$ πόσα; $\binom{n}{2}\cdot\binom{4}{2} = \underline{\underline{3n(n-1)}}$

$i > k$ $\qquad$ πού το X_i

5) i,j,k,l όλα διαφ. $\quad \mathbb{E}(X_i X_j X_k X_l) = \mathbb{E}X_i\,\mathbb{E}X_j\,\mathbb{E}X_k\,\mathbb{E}X_l = 0$

$X_1 X_2 X_1 X_2 \qquad i=1,\ k=2$

$i=2 \quad k=1$