Let $\Omega \subset I\!\!R^n$ be an open domain that contains the origin. We find conditions on the potential V which ensure the nonexistence of positive $H^1(\Omega)$ solutions for linear elliptic problems with Hardy-type potentials. For instance, we prove the nonexistence of nontrivial solutions in $H^1(\Omega)$ for the equation

$$-\Delta u = \frac{(n-2)^2}{4} \frac{u}{|x|^2} + bVu$$

The results depend on an integral assumption on the potential V. We also give an example establishing that this integral assumption on V is optimal.