

Construction of sampling sets and interpolation sets near the critical density

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Abstract

Given a relatively compact Borel set $\Omega \subset \mathbb{R}^n$, the Paley Wiener space PW_Ω consists on all square integrable functions with Fourier transform supported on Ω . In the celebrated paper [3], Landau establishes necessary conditions satisfied by sampling and interpolation sets in terms of the so called lower and upper Beurling densities, \mathcal{D}^- and \mathcal{D}^+ , respectively. These conditions are the following:

- (i) A sampling set Λ for PW_Ω satisfies $\mathcal{D}^-(\Lambda) \geq |\Omega|$;
- (ii) An interpolation set Λ for PW_Ω satisfies $\mathcal{D}^+(\Lambda) \leq |\Omega|$.

In [2] Gröchenig, Kutyniok, and Seip extended the concept of Beurling densities to the setting of a locally compact abelian group G , and proved a generalization of Landau's theorem. In the group setting, Ω is a subset of the dual group \widehat{G} , and the Lebesgue measure $|\Omega|$ is replaced by the corresponding Haar measure $m_{\widehat{G}}(\Omega)$. However, in this context, the authors left open the problem of the existence of sampling sets and interpolation sets with densities arbitrarily close to the critical one, $m_{\widehat{G}}(\Omega)$. In this talk, we will discuss some techniques to construct such sampling sets and interpolation sets. In particular, this leads to the solution of the problem stated by Gröchenig, Kutyniok, and Seip. This is a joint work with J. Antezana and C. Cabrelli [1].

References

- [1] Agora E., Antezana J., Cabrelli C., *Multi-tiling sets, Riesz bases, and sampling near the critical density in LCA groups*, (2014) Preprint available at <http://arxiv.org/abs/1404.2317>.
- [2] Gröchenig K., Kutyniok G., Seip K., *Landau's necessary density conditions for LCA groups*, *J. Funct. Anal.* 255 (2008) 1831-1850.
- [3] Landau H. J., *Necessary density conditions for sampling and interpolation of certain entire functions*, *Acta Math.* 117 (1967) 37-52.