Uniqueness and regularity conditions for weak solutions of the Navier-Stokes equations

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Let $[0, T)$ with $0 < T \leq \infty$ be a time interval and $\Omega \subseteq \mathbb{R}^3$ a smoothly bounded domain. Consider in $[0, T) \times \Omega$ the non-stationary nonlinear Navier-Stokes equations with prescribed initial value $u_0 \in L^2_\sigma(\Omega)$ and external force $f = \nabla \cdot F$ with $F \in L^2(0, T; L^2(\Omega))$. It is well-known that there exists at least one weak solution of the Navier-Stokes system in $[0, T) \times \Omega$ in the sense of Leray-Hopf. Since we do not know if these solutions are unique it is an important problem to investigate conditions on the data u_0 and f - as weak as possible - to guarantee the existence of a unique strong solution $u \in L^s(0, T; L^q(\Omega))$ satisfying Serrin's condition $\frac{2}{s} + \frac{3}{q} = 1$ with $2 < s < \infty$, $3 < q < \infty$, at least for $T > 0$ sufficiently small. Our optimal conditions are formulated in terms of certain Besov spaces and represent the largest possible class of such (local) strong solutions.

References and Literature for Further Reading

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