Sharp Hardy-Sobolev estimates for fractional Hardy-Schrödinger operators

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Abstract

Fractional Laplacians have been attracting considerable attention, both for their interesting theoretical structure, and for their natural appearance in many practical applications. A great deal of this attention is devoted to improving fractional Hardy inequalities, associated with these operators. We will restrict our discussion to two different operators of this kind, defined on bounded domains: the *spectral* and the *Dirichlet Laplacian*.

More precisely, after a concise discussion on some motivating results, we will establish sharp Hardy-Sobolev type estimates, for the Hardy-Schrödinger operator $\mathcal{L} = (-\Delta)^s - k_{n,s}\frac{1}{|x|^{2s}}$. Here $(-\Delta)^s$ stands for the aforementioned fractional Laplacians defined on a bounded domain of $\mathbb{R}^n$, $s \in (0,1)$, and $k_{n,s}$ is the best constant in the corresponding fractional Hardy inequality.

An important feature of these fractional Laplacians is their nonlocal character, which can be realized as a well known mapping from Dirichlet to Neumann type boundary conditions, via a local problem posed on the extended upper half space. The key point in deriving our results is the equivalence of the original nonlocal problem with the extended local problem, where local variational techniques can be applied.

Our estimates, under a 'ground state' transformation, yield sharp estimates corresponding to certain limiting cases of fractional Caffarelli-Kohn-Nirenberg type inequalities.