TITLES AND ABSTRACTS FOR "COMPLEX AND HARMONIC ANALYSIS 2009"

1. Plenary talks

1. **Dimitris Betsakos**: An extremal problem for the hyperbolic metric on Denjoy domains

Suppose that Ω is a domain in the extended complex plane and assume Ω contains the origin and that the boundary of Ω lies on the interval $[-1, 1]$ and has total length $2m$, $0 < m < 1$. We study the problem of finding the infimum of the density of the hyperbolic metric $\lambda(0, \Omega)$ at the origin among all such domains Ω. We show that if m is sufficiently large, the infimum is attained uniquely for the doubly connected domain which is symmetric in the imaginary axis. This result improves an estimate of A.Yu. Solynin. We also show that for sufficiently small m the above domain is not any more extremal.

2. **Oscar Blasco**: Projections of the Bergman space

We define a set of projections on the Bergman space A^2 parameterized by an affine closed space of a Banach space. This family is defined from an affine space of a Banach space of holomorphic functions in the disk and includes the classical Forelli-Rudin projections. (Joint work with Salvador Perez-Esteva, UNAM, Méjico)

3. **Joaquim Bruna**: Basis of complex sponentials and related problems

The talk will survey results and open problems related to the existence of Riesz basis of exponentials on general sets lying in Euclidean space. We will also consider related problems on frames of exponentials, spanning properties of translates, reproducing kernel Hilbert spaces of entire functions, and generalizations of the above to other settings.

4. **Daniel Girela**: On Blaschke products Bloch functions and normal functions

In this talk we shall discuss a number of questions relating multiplication and division by distinct classes of Blaschke products, Bloch functions and normal functions.

5. **Antonios Melas**: Sharp mixed norm inequalities for dyadic maximal operators

For each $1 < q < p$ we precisely evaluate the main Bellman functions associated with the local $L^p \to L^q$ estimates of the dyadic maximal operator on $\mathbb{R}$. Actually we do that in the more general setting of tree-like maximal operators and with respect to general convex and increasing growth functions. We prove that these Bellman functions are equal to analogous extremal problems for local Hardy-type operators which can be viewed as a symmetrization principle. We also obtain the corresponding Bellman function for the $L \log L \to L^1$ inequality.

6. **Vassilis Nestoridis**: Domains of injective holomorphy

The talk is based on joint work with P. Gauthier.

A planar domain is called a domain of injective holomorphy if it supports an injective holomorphic function, which is not extendable as a holomorphic function.

A domain is called regular if it coincides with the interior of its closure. Such a planar domain is always a domain of injective holomorphy. The same is true for every simply connected domain in the complex plane. We concider the components in the Riemann sphere of the complement of a finitely connected planar domain. If two of them are singletons in the complex plane, then the initial domain is not a domain of injective holomorphy; otherwise, it is a domain of injective holomorphy.

7. **Aristomenis Siskakis**: Integration operators on spaces of analytic functions

We will review properties and results about operators of the form

$$T_g(f)(z) = \int_0^z f(\zeta)g'(\zeta)d\zeta$$

acting on various spaces of analytic functions of the unit disc.

8. **Dragan Vukotic**: An overview of products of Toeplitz operators with small range

In this talk we will discuss the finite products of Toeplitz operators on Hardy and Bergman spaces, especially the questions of when are such products equal to zero or have finite rank. We will survey the results obtained by various authors over the last 10-15 years and will give an outline of our recent solution (jointly with A. Aleman) of the finite rank problem (and thus the zero product problem) on general Hardy spaces.

2. Contributed talks

1. **George Costakis**: Locally topologically transitive operators at zero

We look at operators having interesting dynamical behavior only "near" 0. Such an operator T acting on a Banach space X has the following properties:

(i) for every open neighborhood U_0 of 0 and every open set V there exists some positive integer n such that $T^n(U_0) \cap V \neq \emptyset$.

(ii) For every $x \in X \setminus \{0\}$ there exist an open neighborhood U_x of x and an open set V such that $T^n(U_x) \cap V = \emptyset$ for every $n = 0, 1, 2, \ldots$.

We shall present some naive (and also some less trivial) examples of operators exhibiting such a behavior, using some deep recent results on hypercyclic operators due to several authors and we shall discuss some related open questions. This is joint work (on progress) with A. Manoussos.

2. **Santiago Diaz-Madrigal**: Generalized Loewner chains

In this talk, we introduce a general version of the notion of Loewner chains which allow a new and unified treatment of the radial and chordal variant of the Loewner differential equation and the theory of semigroups of analytic functions. In this very general setting, we establish a deep correspondence between these chains, certain evolution families and some particular non-autonomous dynamical systems. Among other things, we show that such a correspondence is essentially one-to-one. In a similar way as in the classical Loewner theory, we also prove that these chains are solutions of a certain partial differential equation which resembles (and includes as a very particular case) the classical Loewner-Kuffarev PDE.

3. **Cristobal Gonzalez**: Univalent functions in Hardy spaces in terms of their arc-length growth

In 1962, Pommerenke gave the following characterization: *for f univalent in the unit disk and $0 < p < 2$, $f \in H^p$ iff $\int_0^1 M_1^p(r, f')dr < \infty$*. In fact, this characterization was done for multivalent functions and, in this setting, he proved that the result cannot be extended to $p > 2$.

Another characterization of univalent functions in H^p (for any $p > 0$) is given by requiring that $\int_0^1 M_\infty^p(r, f)dr < \infty$. A more recent characterization (Baernstein, Girela, Peláez, 2004) requires $\int_0^1 (1 - r)^{p-1} M_p^p(r, f')dr < \infty$.

It is then natural to ask whether the result of Pommerenke, in the univalent case, is true for all $p > 0$. We shall see that the result holds true for p slightly larger than 2, but is false for large p.

This is part of a joint work with J.A. Peláez.

4. **Sigrid Heineken**: On the structure of sampling spaces

We consider the sampling problem in general shift invariant spaces. We describe the structure of sampling spaces and find characterizations of functions that belong to a sampling space. Examples are given.

5. **Denis Khristoforov**: On uniform approximation of elliptic functions by means of Pade approximants

Let f be elliptic function analytic at infinity. According to Stahl theorem, diagonal Pade approximants $[n/n]_f$ developed at $z = \infty$ converge to $f(z)$ in capacity in a special kind domain $D_f = \overline{\mathbb{C}} \setminus S$, where S is the minimal capacity set consisting of two analytic curves and f can be continued throughout D_f in a meromorphic way.

The main difficulty for uniform convergence of $[n/n]_f$ in D_f is the existence of spurious poles of Pade approximants, which do not correspond to singularities of $f(z)$. The work presents constructive procedure for uniform approximation of f for the case of general position. The behaviour of spurious poles and asymptotics of remainder function are studied, so a new sequence of piecewise rational approximants $\hat{f}_n$ is proposed.

Sequence $\{\hat{f}_n\}$ consists of adjacent approximants $[n/n]_f$ and $[n + 1/n + 1]_f$ and is proved to uniformly approximate $f(z)$ in D_f:

$$f(z) = \hat{f}_n(z)[1 + o(1)]\,, \quad z \in D_f\,, \quad n \to \infty\,.$$

6. **Maria Martin**: Norm attaining composition operators on the Bloch spaces

We prove that every composition operator C_ϕ on the Bloch space modulo constant functions attains its norm and we characterize the norm attaining composition operators on the little Bloch space (modulo constant functions). We also prove, among other results, that the extremal functions for the norm of C_ϕ are never unique.

7. **Bruce O'Neill**: Smoothness of star measures of convex functions

Let f be an element of S, the familiar class of normalized univalent functions on the unit disc. If f is starlike, then the real part of $zf'(z)/f(z)$ is positive, and hence is given by a unique positive unit measure on the boundary of the disc.

By appeal to an elegant geometric criterion of Pommerenke we show that if, in addition, f is convex, then this measure is quite tame: its Lebesgue decomposition consists of at most one point mass, necessarily of mass at most 1/2, and the remainder is absolutely continuous. In particular, there is no continuous singular term.

Sharp examples are given, and related problems are discussed.

8. **Jordi Pau**: Integration operators on weighted Bergman spaces

For an analytic function g on the unit disc, we consider the operators

$$T_g f(z) = \int_0^z f(\zeta)\, g'(\zeta)\, d\zeta.$$

We describe the boundedness and compactness of T_g on Bergman spaces with exponential weights, answering a question posed by Aleman and Siskakis in 1997.

9. **Stamatis Pouliasis**: Superharmonicity and harmonicity properties of condenser energy under dilation

Consider a domain D in the extended complex plain C and a compact subset K of D such that $D \setminus K$ is connected. If z is a complex number, the dilation of D is the set $D_z = \{zw : w \in D\}$. If $E(z)$ is the energy of the condenser with plates K and $C \setminus D_z$, for all z such that K is a subset of D_z, then E is superharmonic. E is harmonic if and only if $Int(K)$ and D are approximately concentric disks with center at the origin.

10. **Romi Shamoyan**: Some remarks on Cauchy-Stieltjes type integrals and spaces of their multipliers in U^2

We will present various properties of holomorphic $f = f(z_1, z_2)$ functions in U^2 which belong to classical analytic classes (Hardy,Bergman,etc) but only by one variable (first z_1 or second z_2) and can be represented via Cauchy-Stieltjes type integrals by another variable (or belong to spaces of their multipliers.) We will follow arguments and propositions from [1],[2] and combining them with numerous known results on classical analytic classes in the unit disk (or bidisk) we will provide various properties of such functions.

[1] S.A.Vinogradov,Proceedings of Seventh Winter School,Drogobich,1974.
[2] S.A.Vinogradov,M.Goluzina,V.Havin,Zapiski LOMI,v.19,1970.

11. **Nikolaos Tsirivas**: Examples of extended universal series

In this talk we present three new non-trivial examples of the extended abstract theory of universal series that has been developed recently by D. Hadjiloucas. These examples avoid the Cartesian topology and are examples of universal Taylor series in simply connected domains in the context of the above mentioned extended abstract theory of universal series.

12. **Vagia Vlachou**: Common hypercyclic vectors for Birkhoff-type operators

Let $\{\lambda_n\}_{n\in\mathbb{N}}$ be a sequence of complex numbers such that: $\lim_{n\to\infty} \lambda_n = \infty$.
It is known that for every $\zeta \in \mathbb{C}\setminus\{0\}$, there exists an entire function f such that the set $\overline{\{f(z+\lambda_n\zeta);\ n\in\mathbb{N}\}}$ is dense in $H(\mathbb{C})$ endowed with the topology of uniform convergence on compacta.
In this talk we shall discuss the following question: Does there exist an entire function f such that the set $\overline{\{f(z+\lambda_n\zeta);\ n\in\mathbb{N}\}}$ is dense in $H(\mathbb{C})$, for every $\zeta \in \mathbb{C}\setminus\{0\}$? The answer to this question depends heavily on the sequence $\{\lambda_n\}_{n\in\mathbb{N}}$.
This is a work in progress with G.Costakis and N.Tsirivas.

13. **Marisa Zymonopoulou**: Isometric embeddings, Absolute sums of normed spaces and Positive definite distributions

Positive definite norm dependent functions are closely related to isometric embedding of normed spaces into L_p. The concept of isometric embedding of finite dimensional spaces into L_p has been extended analytically for negative values of p. The advantage is that we can extend several facts from the theory of L_p spaces, for $p > 0$, and get new geometric results.

Let $X = (\mathbb{R}^n, \|\cdot\|_X)$, $Y = (\mathbb{R}^m, \|\cdot\|_Y)$ be two normed spaces and $0 < r \leq 2$. We consider the r-sum, $X \oplus_r Y$, that is defined as the space of pairs $\{(x, y), x \in X, y \in Y\}$ equipped with the norm $|||(x, y)||| = \left(\|x\|_X^r + \|y\|_Y^r\right)^{1/r}$. We use Fourier analytic tools to show that isometric embedding of the space $X \oplus_r Y$ into L_p, with $p \in (-n - m, r - m)$, implies that X embeds into L_{p+m}.

Some other results will be presented including the case where the norm $|||\cdot|||$ is substituted by any absolute normalized norm that satisfies mild conditions.

This is a work in progress with Nigel Kalton.