Talks for

Analytical and Combinatorial Methods in Number Theory and Geometry 2007

1. **Iskander Aliev**
Covering problems and the Frobenius number

In the talk we will discuss recent lower and upper estimates for the Frobenius number obtained by applying results from geometry of numbers.

This is a joint talk with Peter Gruber (TU Wien).

2. **Paulius Drungilas**
Unimodular roots of reciprocal Littlewood polynomials

3. **Ramūnas Garunkštis**
On the zeros of the Hurwitz zeta-function

We consider asymptotics for the sum of values of the Hurwitz zeta-function $\zeta(s,\alpha)$ taken at the nontrivial zeros of the Riemann zeta-function $\zeta(s) = \zeta(s,1)$ when the parameter α either tends to $1/2$ or 1; the case $\alpha = 1/2$ is of special interest since $\zeta(s,1/2) = (2^s - 1)\zeta(s)$. Besides, we present several computer plots which reflect the dependence of zeros of $\zeta(s,\alpha)$ on the parameter α. Inspired by these plots we call a zero of $\zeta(s,\alpha)$ stable if its trajectory starts and ends on the critical line as α varies from 1 to $1/2$.

4. **Martin Huxley**
Configurations of lattice points inside a closed curve

A shape is represented on a computer screen by the set of integer points inside the shape. As the shape moves, the set of integer points changes. In joint work with Zunic (Exeter), Kolountzakis (Crete) and Konyagin (Moscow) we estimate the number of configurations (up to translation), either with the true size fixed, or with the screen size (number of integer points) fixed. We get asymptotic formulas if the shape cannot have too many integer points on the boundary. The most interesting shape is the circle, which can have many integer points on its boundary. We get some but not all of the asymptotic formulae in the case of the circle using careful arguments involving the factorisation of Gaussian integers $a + ib$.

5. **Mihalis Kolountzakis**
Translational tiling in Fourier space

A set in an abelian group is said to tile the group by translation if it can be translated around so as to cover every element of the group exactly once. In this talk I will review results obtained in problems of translational tiling in the last 10-15 years. A common characteristic of these is the expression of the tiling condition in Fourier space as a condition on the zero-set of the Fourier

Transform of the tile and the distributions this zero-set can support. The problems will include: structure of tilings, the Steinhaus tiling problem, and the problem of spectral sets. This last class of sets was conjecturally (Fuglede) the same as the sets which tile by translation. This conjecture has recently been refuted down to dimension 3. Several open problems will be stated.

6. **Nikolay Moshchevitin**
Diophantine approximations with lacunary and sublacunary sequences

A sequence $\{t_j\}$, $j = 1, 2, 3, ..$ of positive real numbers is defined to be lacunary if for some $M > 0$ one has

$$\frac{t_{j+1}}{t_j} \geq 1 + \frac{1}{M}, \ \ \forall j \in \mathbb{N}. \tag{1}$$

A. Khintchine in 1924 proved, that there exists a real number α and positive γ such that for any sequence under the condition (1) the following statement is valid:

$$||t_n \alpha|| \geq \gamma \ \ \forall n \in \mathbb{N}.$$

We should note that Khintchine's approach enables to prove the existence of real α and positive γ such that

$$||t_n \alpha|| \geq \frac{\gamma}{(M \log M)^2} \ \ \forall n \in \mathbb{N}.$$

50 years later (in 1974) Erdös conjectured that for any lacunary sequence there exists real α such that the set of fractional parts $\{\alpha t_j\}$, $j \in \mathbb{N}$ is not dense in $[0, 1]$. Khintchine's theorem cited above gives a positive answer to Erdös' conjecture. But Khintchine's result was forgotten. An answer to Erdös' conjecture was published by A. Pollington and B. de Mathan. Some quantitative improvements were due to Y. Katznelson, R. Akhunzhanov and N. Moshchevitin and A. Dubickas. The best known quantitative estimate is due to Y. Peres and W. Schlag. The last authors proved that with some positive constant $\gamma > 0$ for any sequence $\{t_j\}$ under consideration there exists a real number α such that

$$||\alpha t_j|| \geq \frac{\gamma}{M \log M}, \ \ \forall j \in \mathbb{N}.$$

Y. Peres and W. Schlag use an original approach connected with the Lovasz local lemma.

From another hand R. Akhunzhanov and N. Moshchevitin generalized the constructions to sublacunary sequences. For example for a sequence $\{t_j\}$ under condition

$$\frac{t_{j+1}}{t_j} \geq 1 + \frac{\gamma}{n^\beta}, \ \ \forall j \in \mathbb{N}, \gamma > 0, \beta \in (0, 1/2]$$

they proved the existence of real irrational α such that

$$\liminf_{n \to \infty} \left(||t_n \alpha|| \times n^{2\beta} \right) > 0.$$

Another application from deals with the sequence of naturals of the form $2^m 3^n$, $m, n \in \mathbb{N} \cup \{0\}$.

In the lecture some generalizations Peres-Schlag's result will be announced.

7. **Michael Papadimitrakis**
Oscillatory Singular Integrals with Polynomial Phase

We shall prove an optimal estimate for the measure of the sublevel set of a polynomial $p(x)$ and show how this, together with the Van der Corput lemma, can be used to find the optimal estimate

$$\Big| P.V. \int_{-\infty}^{+\infty} e^{ip(x)} \frac{dx}{x} \Big| \leq c \log d,$$

where c is an absolute positive constant and d is the degree of $p(x)$. This is joint work with Y. Parissis.

8. **Andrei Raigorodskii**
On Ramsey type problems in combinatorial geometry

In our talk we will discuss various problems that belong simultaneously to combinatorial geometry and Ramsey theory. Among them: the Nelson-Hadwiger problem on the chromatic numbers of metric spaces and the Erdős-Szekeres problem on finding convex polygons in sets in general position.

9. **Szilárd Révész**
Oscillation of the remainder term in the Beurling prime number formula

Arne Beurling generalized the prime number theorem to the rather general situation when the role of primes are taken over by some arbitrary reals, and integers are simply the reals of the freely generated multiplicative subgroup of the primes given. If the "number of integers from 1 to x" function takes the form $N(x) = x + O(x^a)$, with $a < 1$, then the corresponding Beurling zeta function has a meromorphic continuation to the halfplane to the right of a. Location of zeroes between the real part $= 1$ and real part $= a$ lines are then crucial to the oscillation of the prime number formula, as is well-known in the classical case. The lecture aims at an analogous description of these relations even in the generality of Beurling prime distribution. In particular, we explain what (relatively mild) conditions can ensure that the most well-known classical zero density estimates carry over to the Beurling zeta function, too.

10. **Wadim Zudilin**
Ramanujan's formulae for $1/\pi$ and their generalisations

In 1914 S. Ramanujan recorded a list of 17 series for $1/\pi$, which produces rapidly converging (rational) approximations to π. I will survey the methods of proofs of Ramanujan's formulae and indicate recently discovered generalisations, some of which are not yet proved.