# TITLES AND ABSTRACTS FOR HARP 2006

## 1. Plenary talks

1. **Jonathan Bennett**: Multilinear Kakeya inequalities

We will discuss an approach to certain multilinear Kakeya-type inequalities in $\mathbb{R}^d$ at various levels of multilinearity. Applications will be described to multilinear analogues of some classical problems in harmonic analysis, with particular emphasis on the restriction conjecture for the Fourier transform. A certain "self-improving" property leading to diffeomorphism-invariant analogues of these results will also be presented. This is joint work with A. Carbery and T. Tao.

2. **Michael Christ**: The nonlinear Schrodinger equation, multilinear operators, and (pseudo-)Feynman diagrams

The Cauchy problem for the one-dimensional periodic cubic nonlinear Schrödinger equation is $iu_t + u_{xx} + \omega|u|^2u = 0$ with initial datum $u(0, x) = f(x)$, where $x \in \mathbb{R}/2\pi\mathbb{Z}$, $t \in \mathbb{R}$, and $\omega = \pm 1$. This problem is wellposed in $H^0 = L^2$, but not in $H^s$ for negative $s$.

We discuss solvability in the space of distributions whose Fourier coefficients belong to $\ell^p$, for $1 \le p < \infty$; for $p > 2$, these spaces scale like $H^s$ for negative $s$. The novelty lies mainly in the analysis, and the theorem is to an extent merely an excuse for doing the proof.

The method is to prove directly that the mapping from datum to solution (at a fixed time $t > 0$) is analytic, by formally expanding the solution in a power series, in which each term is a multilinear operator of arbitrary degree. These terms are indexed by a class of trees analogous to Feynman diagrams, and are rather complicated. $\ell^p$ inequalities are established for these operators, and the infinite series is summed, to control the solution. A bit of elementary number theory is used.

The nonuniqess of solutions in $C^0(H^s)$ for certain exponents $s$, and in the above spaces for $p > 2$, will be discussed briefly.

3. **Guy David**: Minimal and almost-minimal sets in $\mathbb{R}^n$

The motivation for this lecture is potential existence results for closed sets $E \subseteq \Omega$ that minimize a functional like

$$J(E) = \int_E h(x) dH^d(x)$$

under some topological constraints on $E$ , where $h$ is a continuous function on the domain $\Omega \subseteq \mathbb{R}^n$, with $C^{-1} \le h \le C$ on $\Omega$, $H^d$ is the Hausdorff measure (or $d$-dimensional surface measure), and the constraints also force $E$ to be at least $d$-dimensional. Think about the Plateau problem for soap films in $\mathbb{R}^3$. Minimizers for $J$ are typical examples of almost-minimal sets.

A scheme to study such existence problems without currents would use lowersemicontinuity results for the restriction of $H^d$ to almost-minimal sets and a good local description of almost-minimal sets. Such a scheme may well work in some cases, at least when $d = 2$ and $n = 3$, because of Jean Taylor's celebrated regularity result on the local regularity of two-dimensional almst-minimal sets in $\mathbb{R}^3$.

We shall either try describe the scheme above, or spend more time on a proof of J. Taylor's regularity result.

4. **M. Burak Erdogan**: Averages of the Fourier transform of measures and some applications

We call a Borel probability measure $\mu$ in $\mathbb{R}^d$ $\alpha$-dimensional, $\alpha \in (0,d)$, if

$$I_\alpha(\mu) := \int \frac{d\mu(x)d\mu(y)}{|x-y|^\alpha} < \infty.$$

The $L^2$ spherical averages of the Fourier transform of such measures, $\|\widehat{\mu}(R\cdot)\|_{L^2(S^{d-1})}$, always decay like $R^{-\beta}$ as $R$ tends to $\infty$ for some $\beta = \beta(\alpha) > 0$. In this talk, we discuss some recent results on the lower bounds on $\beta$. These results have applications in geometric measure theory and linear PDE's.

5. **Alex Iosevich**: Analysis, combinatorics and number theory of the Mattila integral

Let $P$ denote a collection of points and $L$ a collection of geometric objects. Counting the number of incidences between the elements of $P$ and $L$ is a classical problem that arises in several areas of mathematics and computer science. We shall refer to the $L^2$ norm, appropriately defined, of this incidence function, as the Mattila integral, introduced by P. Mattila in the study of the Falconer distance problem. We shall discuss analytic, combinatorial and number theoretic aspects of this quantity in Euclidean space and vector spaces over finite fields. Particular emphasis will be placed on number theoretic consequences of analytic inequalities and the restrictions that these connections imply.

6. **Sergei Konyagin**: Additive properties of product sets in fields of prime order

Let $p > 2$ be a prime, $\mathbb{Z}_p$ be the field of the residues modulo $p$. For a set $X \subset \mathbb{Z}_p$ and $k \in \mathbb{N}$ we denote

$$kX = \{x_1 + \cdots + x_k : x_1, \ldots, x_k \in A\},$$
$$X^k = \{x_1 \ldots x_k : x_1, \ldots, x_k \in A\}.$$

If $\varepsilon > 0$, $|A| > p^{1/(k-\varepsilon)}$, then $KA^k = \mathbb{Z}_p$ where $K \ll 4^k \log(2 + 1/\varepsilon)$. This result has been established by A. Glibichuk and the speaker.

7. **Michael Lacey**: On the Small Ball Problem in Three Dimensions

We prove the following inequality, in which $h_R$ denotes a Haar function adapted to a dyadic rectangle $R \subset [0,1]^3$, normalized in $L^\infty$ norm. For all integers $n \geq 1$

$$2^{-n} \sum_{|R|=2^{-n}} |a_R| \lesssim n(\log n)^{-1/2} \Big\| \sum_{|R|=2^{-n}} a_R h_R(x) \Big\|_\infty .$$

This is a logarithmic improvement over a trivial estimate. It is conjectured that the $\log n$ term can be replaced by $n$. (So there is a long way to go to prove the conjecture.) Our proof is a simplification and extension of a famous (and famously difficult) argument of Jozef Beck. It is related to questions in Approximation theory, inequalities for the Brownian sheet, and irreqularities of distributions. Joint work with Dmitry Bilyk.

8. **Pertti Mattila**: Boundedness and convergence of singular integrals with relations to rectifiability

I discuss singular integrals with respect general measures. One of the main questions is: does boundedness imply almost everywhere convergence and in what sense? For m-dimensional (m a positive integer) measures this is related to the question: does boundedness or convergence imply nice geometric properties such as rectifiability? These questions are well understood for the Cauchy kernel 1/z in the complex plane due to Melnikov's identity relating it to the Menger curvature of triples of points. But they are very much open for example for the Riesz kernels in higher dimensions. i

9. **Giancarlo Mauceri**: A BMO space for the Ornstein–Uhlenbeck operator

In this talk I shall describe recent joint work with S. Meda on a space of functions of bounded mean oscillation over the measured metric space $(\mathbf{R}^d, \rho, \gamma)$, where $\rho$ denotes the Euclidean distance and $\gamma$ the Gauss measure. This $BMO(\gamma)$ space plays for the Ornstein-Uhlenbeck operator the same rôle that the classical Calderòn-Zygmund theory plays for the Laplacian on $(\mathbf{R}^d, \rho, \lambda)$, where $\lambda$ is the Lebesgue measure. We show that if $p$ is in $(2, \infty)$, then $L^p(\gamma)$ is an intermediate space between $L^2(\gamma)$ and $BMO(\gamma)$, and that an inequality of John–Nirenberg type holds for functions in $BMO(\gamma)$. We also show that $BMO(\gamma)$ is the dual of an atomic $H^1(\gamma)$ space. As an application, we show that certain singular integral operators related to the Ornstein–Uhlenbeck operator, which are unbounded on $L^1(\gamma)$ and on $L^\infty(\gamma)$, turn out to be bounded from $H^1(\gamma)$ to $L^1(\gamma)$ and from $L^\infty(\gamma)$ to $BMO(\gamma)$.

10. **Alan McIntosh**: Hardy spaces of differential forms on Riemannian manifolds

Let $M$ be a complete Riemannian manifold. Assuming the doubling condition on the volume of balls, we define Hardy spaces $H^p$ of differential forms on $M$ and give various characterizations of them, including a molecular decomposition. As a consequence, we derive the $H^p$-boundedness for Riesz transforms on $M$, generalizing previously known results. Further applications, in particular to $H^\infty$ functional calculus and Hodge decomposition, are given. This is joint work with Pascal Auscher and Emmanuel Russ.

11. **Antonis Melas**: Some sharp estimates for maximal operators

We will present certain best possible inequalities for maximal operators. The first concerns the best possible constant in the weak type inequality for the centered one dimensional Hardy-Littlewood maximal function. Next we will give the exact Bellman function related to the dyadic maximal operator and its generalizations as well as the corresponding function related to the dyadic Carleson imbedding theorem.

12. **Frank Merle**: Global existence result for $H^1$ critical focusing NLS

We will discuss sharp result for global existence for the $H^1$ critical focusing NLS and application to the blow-up problem.

13. **Detlef Müller**: A necessary condition for local solvability of linear differential operators with double characteristics

Consider a linear differential operator $L$ with smooth coefficients defined, say, in an open set $\Omega \subset \mathbb{R}^n$. Assume the principal symbol $p_k$ of $L$ vanishes to second order at $(x_0, \xi_0) \in T^*\Omega \setminus 0$, and denote by $Q_{\mathcal{H}}$ the Hessian form associated to $p_k$ on $T_{(x_0,\xi_0)}T^*\Omega$. The main result I am going to present states that (under some rank conditions and some mild additional conditions) a necessary condition for local solvability of $L$ at $x_0$ is the existence of some $\theta \in \mathbb{R}$ such that $\Re(e^{i\theta} Q_{\mathcal{H}}) \geq 0$. By means of Hörmander's classical necessary condition for local solvability, the proof is reduced to the following question:

Suppose that $Q_A$ and $Q_B$ are two real quadratic forms on a finite dimensional symplectic vector space, and let $Q_C := \{Q_A, Q_B\}$ be given by the Poisson bracket of $Q_A$ and $Q_B$. Then $Q_C$ is again a quadratic form, and we may ask: When can we find a common zero of $Q_A$ and $Q_B$ at which $Q_C$ does not vanish?

14. **Alexander Olevskii**: Sampling of signals with bounded spectra

Let $S$ be a bounded set on $\mathbb{R}$, $\Lambda$ be a real sequence. Consider a signal $f$ with spectrum in $S$. When it can be reconstructed by its values on $\Lambda$?

Classical results in the area are due to Beurling, Malliavin and Landau. I will focus on some new aspects, in particular: is it possible to define a universal sampling $\Lambda$ of given density, which admits reconstruction of all signals with spectra of small measure? Recent results on this problem, joint with Ulanovskii, will be discussed.

15. **Fulvio Ricci**: Analysis of Hodge Laplacians on the Heisenberg group

In this joint work with D. Müller and M. Peloso, we decompose the space of $L^2$-differential $k$-forms on the Heisenberg group under the action of the $U(n)$-invariant riemannian Hodge Laplacian. We obtain a finite decomposition into invariant subspaces, and a unitary transformation of each of them, which transforms the Laplacian into an appropriate $U(n)$-invariant differential or pseudo- differential operator acting on functions. This allows to obtain an Mihlin-Hörmander $L^p$-multiplier theorem for Hodge Laplacians.

16. **Christoph Thiele**: Variants of Carleson's theorem and applications to ergodic theory

Carleson's theorem on almost everywhere convergence on Fourier series is related to various convergence theorems on ergodic averages. In particular we improve on the exponents in Bourgain's Return times theorem. The work is joint with Ciprian Demeter, Michael Lacey and Terence Tao.

17. **Ana Vargas**: Partial recovery of a potential on inverse scattering

We consider the generalized eigenfunctions of the Schrödinger operator in $\mathbb{R}^n$,

$$(-\Delta + q(x))u = k^2 u,$$

where $u = u_i + u_s$, $u_i = e^{ik\theta\cdot x}$ (incoming plane wave) and $u_s$ satisfies the outgoing Sommerfeld radiation condition. The potential $q$ will be assumed to be compactly supported (or decaying at infinity). It is well known that,

$$u_s(k,\theta,x) = \left(\tfrac{k}{|x|}\right)^{\frac{n-1}{2}} e^{ik|x|} u_\infty\left(k,\theta,\tfrac{x}{|x|}\right) + o(|x|^{\frac{1-n}{2}})$$

where $u_\infty(k,\theta,\omega) = \big(q(u_i+u_s)\big)\widehat{\ }(k\omega) = \widehat{q}(k(\omega-\theta)) + \big(qu_s\big)\widehat{\ }(k\omega)$ is the "far field pattern" or "scattering amplitude". The main questions in inverse scattering are

Can we recover $q$ from $u_\infty$? Does $u_\infty$ contain all the information about the singularities of the potential $q$?

We will discuss some recent results on the second question, joint work with J.A. Barceló, D. Faraco and A. Ruiz.

18. **Luis Vega**: Lower bounds for dispersion in Schrodinger equations

The bulk of the talk will be a recent work with L .Escauriaza, C. Kenig and G. Ponce which generalizes a well known result of Hardy on lower bounds for the gaussian decay of a function and its Fourier transform. These are results about dispersion in the spatial variable for solutions of Schrodinger equations. With respect to dispersion in the temporal variable I shall mention some joint work with N. Visciglia.

19. **Jim Wright**: Discrete Averaging Operators

We look at discrete analogues of averaging operators along curves and surfaces. In the finite field setting we use a simple Littlewood-Paley decomposition to obtain sharp $L^p - L^q$ mapping properties for these operators. Passing from finite fields to a finite ring of integers mod $N$ certain degeneracies can occur. By using a more complicated Littlewood- Paley decomposition based on $N$ we obtain nearly sharp estimates for certain operators associated to plane curves. Maximal operators are also examined.

## 2. Contributed talks

1. **Nils Byrial Andersen**: Paley–Wiener theorems for Fourier series

Consider the Fourier series of a function on the circle (or a $2\pi$ periodic function). We characterize the functions with compact support (strictly contained inside the circle) via a condition on the Fourier coefficients.

2. **Valeria Banica**: Scattering for the Schrdinger equation on the hyperbolic space

We study the scattering properties of the nonlinear Schrdinger equation posed on the hyperbolic space. The results are quite different from the ones in the euclidean context, showing the important influence of the geometry on the dynamics of the equation.

3. **Nadine Badr**: Real Interpolation of Sobolev Spaces

Do the Sobolev spaces $W_p^1$ form a real interpolation scale for $1 \leq p \leq \infty$? In $\mathbb{R}^n$, Devore and Scherer proved in 1979 it is the case. This was reproved later differently by Bennett and Sharpley. The aim of my talk is to provide a positive answer for Sobolev spaces on some class of complete non-compact Riemannian manifolds. Namely for those manifold with local doubling property and local $L^q$ Poincaré inequalities and with the restriction $q \leq p \leq \infty$. The method of proof applies on metric measure spaces, Carnot-Caratheodory spaces, Lie groups, weighted Sobolev spaces with the appropriate definition of $W_p^1$.

4. **Besma Ben Ali**: Maximal inequalities and Riesz transform estimates on $L^p$ spaces for Schrödinger operators with nonnegative potentials

We present some new $L^p$ estimates for Schrödinger operators $-\Delta + V$ on $\mathbb{R}^n$ and their square roots. We assume reverse Hölder estimates on the potential. Our main tools are improved Fefferman-Phong inequalities and reverse Hölder estimates for weak solutions of $-\Delta + V$ and their gradients.
This is a joint work with Pascal Auscher.

5. **Ron Blei**: The Grothendieck Inequality Revisited

We prove the following counterpoint to a result by Kashin and Szarek (cf. *Theorem 1, C. R. Acad. Sci. Paris, Ser. I, 1336 (2003) 931-936*):

There exists a map $\phi$ from (the infinite-dimensional Euclidean space) $\ell^2$ into $L^\infty([0,1])$ such that for all real-valued vectors $\mathbf{x}$ and $\mathbf{y}$ in $\ell^2$

$$\|\phi(\mathbf{x})\|_{L^\infty} \leq K\|\mathbf{x}\|_2,$$

$$\sum_j \mathbf{x}(j)\mathbf{y}(j) = \int_{[0,1]} \phi(\mathbf{x})\phi(\mathbf{y})dt,$$

where $K > 0$ is a universal constant.

6. **Bruno Bongioanni**: Hermite-Sobolev spaces

Sobolev spaces associated to the Hermite operator are defined by means of the appropriate potentials. As in the classical case, we see that the definition coincides with the natural definition of integer order. The Hermite Sobolev spaces are natural domains for boundedness of the associated Hermite-Riesz transforms and also can be used for studying regularity of solutions of the associated Schrdinger equation.

7. **Marcin Bownik**: Anisotropic Triebel-Lizorkin spaces

We introduce and develop anisotropic Triebel-Lizorkin spaces associated with general expansive dilations and doubling measures on $\mathbb{R}^n$ with the use of wavelet transforms. This study extends the isotropic methods of dyadic $\varphi$-transforms of Frazier and Jawerth to non-isotropic settings.

In the close analogy with the isotropic theory, we show that weighted anisotropic Triebel-Lizorkin spaces are characterized by the magnitude of the wavelet transforms in appropriate sequence spaces. We also introduce non-isotropic analogues of the class of almost diagonal operators and we obtain atomic and molecular decompositions of these spaces.

8. **Andrea Carbonaro**: Functional calculus for some perturbations of the Ornstein–Uhlenbeck operator

In a recent paper Carcia-Cuerva, Mauceri, Meda, Sjogren and Torrea have shown that the symmetric finite dimensional Ornstein– Uhlenbeck operator has a bounded holomorphic functional calculus on $L^p$ in the sector of angle $\arcsin|1-2/p|$, $1<p<\infty$. We prove a similar result for some perturbations of the Ornstein–Uhlenbeck operator.

9. **Andrew Comech**: Global attractor for the Klein-Gordon equation with a nonlinearity supported at a point

We consider the long-time asymptotics of all finite energy solutions to the Klein-Gordon equation in one dimension, with the nonlinearity concentrated at a point. The main result is that the attracting set of any finite energy solution consists of "nonlinear eigenfunctions", also known as solitary or standing waves.

The argument is based on application of the Titchmarsh Convolution Theorem to the bound component of the solution, relating Harmonic Analysis with nonlinear PDEs in yet one more way.

The problem is inspired by Bohr's postulate on quantum transitions and Schroedinger's identification of the quantum stationary states to the eigenfunctions of the coupled Maxwell-Schroedinger or Maxwell-Dirac equations.

This is a joint work with Alexander Komech, University of Vienna.

10. **Andrew Comech**: $L^p \to L^q$ estimates for Fourier integral operators near caustics

We derive the relation of $L^p \to L^q$ estimates on the Fourier integral operators and the geometry of the projections from the canonical relation, and find out that the estimates sufficiently far from the $L^p \to L^{p'}$ line are less sensitive to the emergence of caustic points.

11. **Ciprian Demeter**: Convergence and divergence of multilinear averages in ergodic theory

We analyze multilinear combinatorial averages related to multiple recurrence in ergodic theory, and prove their almost everywhere convergence when the coefficient matrix has special rank properties. The positive results are contrasted with some negative ones, when the input functions are in $L^p$ spaces with $p$ sufficiently close to 1. This is joint work with Terence Tao and Christoph Thiele.

12. **Martin Dindos**: $L^p$ Dirichlet problem for elliptic operators with rough coefficients

We present of joint work with J. Pipher and S. Petermichl where we study the Dirichlet $L^p$ solvability of divergence type elliptic operators with (just) $L^\infty$ coefficients. Well know counterexamples show that boundedness and ellipticity is not sufficient for $L^p$ solvability, hence additional condition is required. Usually, some kind of continuity or Dini-type condition is assumed. We instead present a much weaker Carleson type condition that is in some sence "sharp". In particular, we present result that for any $p>1$ if certain Carleson norm of coefficients of the operator is less than $C(p)$ the the $L^p$ problem is solvable.

In addition, if coefficients satisfy vanishing Carleson condition, then the problem is solvable for all $p > 1$. This can be used to show that the $L^p$ Dirichlet problem for the Laplace operator is solvable for all $p > 1$ on Lipschitz domains with the property that *nablaphi* is in the "vmo", where $\phi$ is the Lipschitz function that (locally) determines the boundary. "Vmo" is the space of functions of vanishing mean oscillations.

13. **Anis El Garna**: Orthogonality and Distributional Weight Functions For Dunkl-Hermite Polynomials

In this work, we give a new proof for the orthogonality of the Dunkl-Hermite polynomials $\{H_n^\mu\}_{n\geq 0}$, when the index $\mu > -1/2$, via quasi-monomiality techniques and we show that when $-m-1/2 < \mu < -m+1/2$, where $m$ is a positive integer, $\{H_n^\mu\}_{n\geq 0}$ is an orthogonal set with respect to a sesquilinear form which we express explicitely.
Finally, given a family of polynomials orthogonal with respect to a linear functional $\omega^\mu$ expressed in terms of iteratives of the Dunkl operator on the real line, we treat the problem of extending $\omega^\mu$ to a space of test functions which includes polynomials. As an example, we treat the extension of the weight function with respect to which the Dunkl-Hermite polynomials are orthogonal.

14. **Marton Elekes**: Covering locally compact groups by translates of a compact nullset

Gruenhage asked if it was possible to cover the real line by less than continuum many translates of a compact nullset. (Under the Continuum Hypothesis the answer is obviously negative.) We give an affirmative answer using the well known compact nullset of Erdős and Kakutani. As this set has no analogue in more general groups, we need new ideas to extend the result to locally compact groups.

First we use Pontryagin's duality theory to reduce the problem to three special cases; the circle group, countable products of finite discrete abelian groups, and the groups of $p$-adic integers, and then we solve the problem on these three groups separately. Then we apply these results, representation theory, Lie groups and profinte groups to give a complete (consistent) characterization of those locally compact groups that can be covered by less than continuum many translates of a compact nullset.

15. **Alessandro Figà Talamanca**: Drifted Laplace operators on homogeneous trees and on the hyperbolic half-plane

This talk (based on joint work with E. Casadio Tarabusi)presents, in parallel, results about diffusion with drift on the hyperobolic half plane and a drifted random walk on a homogeneous tree of degree not less than three. The analogies between these two settings are emphasized. Among the results presented are elementary computations, of the Poisson kernels of drifted Laplace operators in the two different contexts, and their relation with stable distributions respectively on the real line and on a local field; a characterization of eigenfunctions of the Laplace operators; the computation of the multiplication operators which intertwine two drifted Laplace operators with unequal drifts.

16. **Troels Johansen**: Tempered Fundamental Solutions for Invariant Differential Operators

Let $X = SO_e(p,q)/SO_e(p-1,q)$, $p,q \geq 2$, $\mathfrak{a}$ a Cartan subspace, and $F_f^{\mathfrak{a}}$ the associated normalized orbital integral of a function $f$ on $X$. Detailed knowledge of the possible discontinuity of $F_f^{\mathfrak{a}}$ at singular points of $\mathfrak{a}$ allow us to construct a tempered $H$-invariant spherical fundamental solution for any nonzero invariant differential operators $D$ on $X$ i.e., a distribution $T$ in the strong dual, $\mathcal{C}'$, of an $L^2$-type Schwartz space of $X$ (with no assumption on $K$-behavior) such that $DT = \delta_e$.

We will also briefly address the surjectivity of such a differential operator on the space of tempered invariant distributions on $X$, in the sense that $D\mathcal{C}' = \mathcal{C}'$.

17. **Norbert Kaiblinger**: Approximation of the Fourier transform from finite samples

An important problem in time-frequency analysis is to determine the dual Gabor window for a given Gabor frame in $L^2(\mathbb{R}^d)$. For the analogous problem in $\mathbb{C}^n$ there exist fast algorithms that make use of the fast Fourier transform. We show how to approximate the dual Gabor window in the continuous-time setting by using methods for $\mathbb{C}^n$. Our approach allows us to work directly with samples of the original Gabor window. The results make use of important properties of Feichtinger's algebra $S_0(\mathbb{R}^d)$, Wiener's Lemma for twisted convolution by Gröchenig and Leinert, and Reiter's Ideal Theorem for Segal algebras. With less effort the same framework allows us to approximate the Fourier transform of a given function, as will be demonstrated.

18. **Tamás Keleti**: On the determination of sets by their triple correlation

Let $G$ be a finite abelian group and $E$ a subset of it. Suppose that we know for all subsets $T$ of $G$ of size up to $k$ for how many $x \in G$ the translate $x + T$ is contained in $E$. This information is collectively called the $k$-deck of $E$. We determine the values of $n$ for which the 3-deck of a subset of the cyclic group $\mathbb{Z}_n$ is sufficient to determine the set up to translation. For this we need nontrivial results about the possible support of the discrete Fourier transform of characteristic functions on $\mathbb{Z}_n$. This is joint work with M. Kolountzakis.

19. **Vladimir Lebedev**: Rate of growth in in Beurling–Helson theorem

Let $A_p(\mathbb{T})$ be the space of the functions $f$ on the circle $\mathbb{T}$ with the sequence of Fourier coefficients $\widehat{f} = \{\widehat{f}(k),\ k \in \mathbb{Z}\}$ in $l^p,\ 1 \le p < 2$. Define the norm in $A_p(\mathbb{T})$ by $\|f\|_{A_p} = \|\widehat{f}\|_{l^p}$. We study the rate of growth of the norms $\|e^{in\varphi}\|_{A_p}$ as $|n| \to \infty$ for $C^1$ smooth real-valued functions $\varphi$. In particular we show that if $\varphi$ is not linear and the derivative $\varphi'$ satisfies Hölder condition of order $\alpha$, then $\|e^{in\varphi}\|_{A_p} \ge c\,|n|^{\frac{1}{p}-\frac{1}{1+\alpha}}$. In the case $\varphi \in C^2$ this implies the earlier known results. For $p = 1$ our estimate is close to be sharp, namely, for every $\alpha,\ 0 < \alpha < 1$, we construct a nonlinear function $\varphi$ with the derivative satisfying Hölder condition of order $\alpha$ and such that $\|e^{in\varphi}\|_{A_1} = O\big(|n|^{\frac{\alpha}{1+\alpha}}(\log|n|)^{\frac{1-\alpha}{1+\alpha}}\big)$. For $p > 1,\ p \ne 1+\alpha$, our estimate is sharp. The same function $\varphi$ satisfies $\|e^{in\varphi}\|_{A_p} = O\big(|n|^{\frac{1}{p}-\frac{1}{1+\alpha}}\big)$, for $p < 1+\alpha$, and $\|e^{in\varphi}\|_{A_p} = O(1)$, for $p > 1+\alpha$. The estimates in $A_p$ imply new results in the study of the following question: if $D$ is a domain in $\mathbb{R}^d$ with $C^1$ -smooth boundary and $\widehat{1_D}$ is the Fourier transform of the indicator function $1_D$, when do we have $\widehat{1_D} \in L^p$?

20. **Nir Lev**: Piatetski-Shapiro phenomenon in the uniqueness problem

We extend to $l^q$ spaces the phenomenon discovered by Piatetski-Shapiro in 1954 : for any $q > 2$ we construct a compact K on the circle, which supports a distribution with Fourier transform in $l^q$, but does not support a measure with this property.

Joint work with A. Olevskii.

21. **Elijah Liflyand**: Hausdorff operators on $H^1$ and BMO

For a wide family of multivariate Hausdorff operators, sufficient conditions for the boundedness of an operator from this family is proved on the real Hardy space and on BMO. By this we extend and strengthen previous results due to K.F. Andersen and F. Móricz. As for accomodation, the situation is still a little bit vague.

22. **Neil Lyall**: Strongly Singular Integrals along curves in $\mathbb{R}^d$

We propose to discuss the regularity of operators of the form

$$T_\gamma f(x) = \text{p.v.} \int_{-1}^{1} H_{A,B}(t) f(x - \gamma(t))dt,$$

along curves $\gamma(t)$ in $\mathbb{R}^d$ which are of finite-type, where

$$H_{\alpha,\beta}(t) = t^{-1}|t|^{-\alpha} e^{i|t|^{-\beta}}$$

is a strongly singular (convolution) kernel in $\mathbb{R}$.

Our results both generalise and extend to higher dimensions those obtained by Chandarana in the plane. This work is joint with Norberto Laghi.

23. **Máté Matolcsi**: Constructions of complex Hadamard matrices via tiling Abelian groups

While studying the spectral set conjecture of Fuglede it became apparent that there is a connection between the existence of complex Hadamard matrices and tiling of Abelian groups. We exploit this connection to obtain new parametric families of complex Hadamard matrices of orders 8 and 12. These families complement the (admittedly incomplete) recent catalogue of complex Hadamard matrices by Zychkowski and Tadej (1). Complex Hadamard matrices have raised recent interest due to their use in quantum-information theory.

(1) W. Tadej, K. Życzkowski: A concise guide to complex Hadamard matrices *preprint.*
(2) M. Matolcsi, J. Réffy, F. Szőllősi: Construction of complex Hadamard matrices via tiling Abelian groups, *preprint.*

24. **Tamás Mátrai**: Norm continuity and resolvent estimates for operator semigroups

According to a fundamental result in the theory of strongly continuous operator semigroups, the asymptotic behavior of a *norm continuous* semigroup is determined by the spectrum of its generator. Therefore the problem of characterizing the norm continuity of semigroups received much attention in the past decades. The particular problem, known as Amnon Pazy's Question in the literature, whether the decay of the resolvent along the imaginary axes is equivalent with the norm continuity of the semigroup in arbitrary Banach spaces was particularly much studied. We answer this question in the negative by constructing a suitable Banach space and an operator semigroup on it. We also discuss a recent result of M. Girardi and L. Weis which characterizes the norm continuity of semigroups in UMD spaces by an $R$-boundedness condition.

25. **Pedro Miana**: $H^\infty$-functional calculus and Mikhlin-type multiplier condition

Let $T$ be a sectorial operator. It is known that the existence of a bounded (suitably scaled) $H^\infty$ calculus for $T$, on every sector containing the positive half-line, is equivalent to the existence of a bounded functional calculus on the Besov algebra $\Lambda^\alpha_{\infty,1}(\mathbb{R}^+)$. Such an algebra includes functions defined by Mikhlin-type conditions and so the Besov calculus can be seen as a result on multipliers for $T$. In this paper, we use fractional derivation to analyse in detail the relationship between $\Lambda^\alpha_{\infty,1}$ and Banach algebras of Mikhlin-type. As a result, we obtain a new version of the quoted equivalence.

26. **Christos Papadimitropoulos**: Abstract theory of universal series and applications

Let $x_1, x_2, \ldots, x_n, \ldots$ be fixed vectors of a topological vector space $X$. A sequence of scalars $\alpha_n$ defines a universal series $\sum_{n=0}^{\infty} \alpha_n x_n$ if the partial sums $\sum_{k=0}^{n} \alpha_k x_k$, $n = 0, 1, 2, \ldots$ are dense in $X$. The sequence $\alpha_n$ may be unrestricted or with the restriction to belong to a specific space of sequences as for example $c_0$, $l_p$, etc. We give a necessary and sufficient condition for existence, genericity, algebraic genericity of universal series. This condition requires a simultaneous double approximation which in several particular cases is guaranteed by well known approximation theorems such as Mergelyan's, Runge's, Weierstrass's, Walsh's theorems, etc. In this way we obtain simplified and unified proofs of most of the classical results as well as of several new ones. In particular we present Menchoff's universal trigonometric series on the torus $T^N$.

This talk is based on the following two papers: (a) Nestoridis, Papadimitropoulos, "Abstract Theory of Universal Series and an Application to Dirichlet series". Comptes Rendus, 2005. (b) Bayart, Grosse-Erdmann, Nestoridis, Papadimitropoulos, "Abstract Theory of Universal Series and Applications", submitted.

27. **Stefanie Petermichl**: AVERAGING, $p-1$, AND POWERS OF THE BEURLING TRANSFORM

The Beurling Ahlfors transform is the Caderon Zygmund operator expressed by convolution against $1/z^2$ in the complex plane. Its significance lies in the fact that it turns z derivatives into $\overline{z}$ derivatives. For some applications, its exact numeric $L^p$ norms are of crucial importance. The $L^p$ norms of the $n$-th powers of the Beurling-Ahlfors transform in dependence of $n$ and $p$ can be estimated via interpolation with the help of a technical endpoint result, though does this proof not give a numeric estimate for the absolute constant that appears. We present a different proof that in some sense is a rotation method for operators with even kernels, delivering a weaker estimate in terms of $n$, but good control on the absolute constant.

28. **Isaac Pesenson**: ANALYSIS OF PALEY-WIENER FUNCTIONS IN $L_p$-NORMS ON NON-COMPACT SYMMETRIC SPACES

By using Bernstein-type inequality we define analogs of spaces of entire functions of exponential type in $L_p(X), 1 \leq p \leq \infty$, where $X$ is a symmetric space of non-compact. We give estimates of $L_p$-norms, $1 \leq p \leq \infty$, of such functions (the Nikolskii-type inequalities) and also prove the $L_p$- Plancherel-Polya inequalities which imply that our functions of exponential type are uniquely determined by their inner products with certain countable sets of measures with compact supports and can be reconstructed from such sets of "measurements" in a stable way.

29. **Vittoria Pierfelice**: NONLINEAR SCHROEDINGER EQUATIONS ON FOUR-DIMENSIONAL COMPACT MANIFOLDS

We prove two new results about the Cauchy problem in the energy space for nonlinear Schroedinger equations on four-dimensional compact manifolds. The first one concerns global wellposedness for Hartree-type nonlinearities and includes approximations of cubic NLS on the sphere. The second one provides, in the case of zonal data on the sphere, local wellposedness for quadratic nonlinearities as well as global wellposedness for small energy data in the Hamiltonian case. Both results are based on new multilinear Strichartz-type estimates for the Schroedinger group.

30. **Elena Prestini**: ALMOST EVERYWHERE CONVERGENCE OF INVERSE FOURIER TRANSFORM

We present a simple and quite general proof of the a.e. convergence of partial integrals of inverse Fourier transforms on Euclidean spaces for square integrable functions with logaritmic Sobolev properties.

31. **Szilárd Révész**: OSCILLATION OF FOURIER TRANSFORMS AND MARKOV-BERNSTEIN INEQUALITIES

Under certain conditions on an integrable function $P$ having a real-valued Fourier transform $\widehat{P}$ and such that $P(0) = 0$, we obtain an estimate which describes the oscillation of $\widehat{P}$ in $[-C\|P'\|_\infty/\|P\|_\infty, C\|P'\|_\infty/\|P\|_\infty]$, where $C$ is an absolute constant, independent of $P$. Given $\lambda > 0$ and an integrable function $\phi$ with a non-negative Fourier transform, this estimate allows us to construct a finite linear combination $P_\lambda$ of the translates $\phi(\cdot + k\lambda)$, $k \in \mathbf{Z}$ such that $\|P'_\lambda\|_\infty > c\|P_\lambda\|_\infty/\lambda$ with another absolute constant $c > 0$. In particular, our construction proves sharpness of an inequality of Mhaskar for Gaussian networks. Joint with Noli N. Reyes and Gino Angelo M. Velasco.

32. **Svetlana Roudenko**: On the concentration phenomenon in the $L^2$-critical NLS equations

We revisit the Bourgain's mass concentration for the $L^2$-critical NLS equation and investigate further the dependence between the size of the concentration window and the growth of the Strichartz norm (joint work with J. Colliander).

33. **Peter Sjögren**: Maximal operators for certain Laguerre functions

The Laguerre functions commonly denoted by $\mathcal{L}_k^\alpha$ form an orthogonal basis on $\mathbb{R}_+$, with respect to Lebesgue measure. Here $\alpha > -1$. The case when $-1 < \alpha < 0$ is rather special, since the $\mathcal{L}_k^\alpha$ then do not belong to $L^p$ for all $1 < p < \infty$. As proved by Macías, Segovia and Torrea, the maximal operator of the associated heat kernel is in that case bounded only on certain $L^p$ spaces. We shall consider this maximal operator in the corresponding multi-dimensional setting.

34. **John Steinberger**: Tilings of the integers with superpolynomial periods

A tiling of the integers is an arrangement of translates of some finite set $A$ of integers such that each integer is contained in exactly one translate. It has long been known that such tilings are periodic, but how long the period can become as a function of the length of $A$ (namely $\max(A) - \min(A)$) is poorly understood. We show that the period can grow faster than any polynomial in the length of A. This improves a 2001 result of Kolountzakis, who showed that the period can grow quadratically with the length of $A$.

35. **Sergey Tikhonov**: Convergence of trigonometric series

We discuss three new convergence criteria (for $p = \infty$, $1 < p < \infty$, and $p = 1$) of belonging of sums of trigonometric series to $L_p$. One-dimensional and multi-dimensional cases are examined. We also study Hardy-Littlewood type theorem for multiple trigonometric and Walsh series in $L_p$ with Muckenhoupt-type weights.

36. **Maria Vallarino**: $H^1$ and $BMO$ spaces on the affine group $ax + b$

Let $G$ be the affine group $ax + b$. We define an atomic Hardy space $H^1$ and a $BMO$ space on the group $G$. We then show that $BMO$ may be identified with the dual space of $H^1$ and that a John-Nirenberg ineguality holds in this setting. We finally prove a real interpolation result for linear operators bounded from $H^1$ to $L^1$ and on $L^p$, for $1 < p < \infty$.

37. **Nicola Visciglia**: Some counterxamples to the Strichartz estimates

For any $\epsilon > 0$ we show the existence of non-negative potentials $V(x)$ that satisfy

$$V(x) \leq \frac{C}{|x|^{2-\epsilon}} \; \forall \; |x| > R,$$

and such that the Strichartz estimates fail for the corresponding solutions to

$$\mathbf{i}\partial_t u - \Delta u + V(x)u = 0, u(0) = f(x).$$

This is a joint work with Michael Goldberg and Luis Vega.

38. **Ahmed Zayed**: Applications of the Directional Wavelet, Ridgelet and Curvelet Transforms in Medical Imaging

It is known that the wavelet transform is very efficient at detecting point singularities, however, when the singularities are of higher dimensions the wavelet transform does not perform very well. The directional wavelet, ridgelet and curvelet transforms were introduced to overcome some of the shortfalls of the wavelet transform.

In this talk we introduce the directional wavelet, ridgelet and curvelet transforms in n dimensions and then discuss some of their applications in medical imaging. In particular, we will compare their performance in texture classification of tissues in computed tomography.