

**Απειροστικός Λογισμός III, χειμερινό εξάμηνο 2016-17.**

**Απαντήσεις πρώτου φυλλαδίου ασκήσεων.**

1. i.  $\frac{\sin 1}{9}$   
ii.  $\frac{1}{(m+1)(n+1)}$   
iii. 0
2.  $\iint_D f(x, y) dx dy = \int_{-1}^1 \left( \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x, y) dy \right) dx = \int_{-1}^1 \left( \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x, y) dx \right) dy$
3.  $\iint_D f(x, y) dx dy = \int_0^1 \left( \int_{x^2}^x f(x, y) dy \right) dx = \int_0^1 \left( \int_y^{\sqrt{y}} f(x, y) dx \right) dy$
4.  $\frac{865}{6}$
5. 0
6.  $3\pi$
7.  $\frac{2}{3}$
8.  $1 + e^{-2}$
9. i.  $\int_0^1 \left( \int_{\sqrt{y}}^1 f(x, y) dx \right) dy$   
ii.  $\int_0^1 \left( \int_{\sqrt{y}}^{3\sqrt{y}} f(x, y) dx \right) dy$   
iii.  $\int_1^e \left( \int_0^{\log y} f(x, y) dx \right) dy$   
iv.  $\int_0^4 \left( \int_{-\sqrt{4-x}}^{\sqrt{4-x}} f(x, y) dy \right) dx$   
v.  $\int_{-3}^3 \left( \int_{|y|}^3 f(x, y) dx \right) dy$   
vi.  $\int_0^1 \left( \int_0^{\sqrt{1-y^2}} f(x, y) dx \right) dy$   
vii.  $\int_0^1 \left( \int_0^{\operatorname{Arccos} y} f(x, y) dx \right) dy$   
viii.  $\int_2^4 \left( \int_1^{y/2} f(x, y) dx \right) dy + \int_4^7 \left( \int_{(y-1)/3}^2 f(x, y) dx \right) dy$   
ix.  $\int_0^{\log 2} \left( \int_1^{e^y} f(x, y) dx \right) dy + \int_{\log 2}^e \left( \int_1^2 f(x, y) dx \right) dy + \int_e^{e^2} \left( \int_{\log y}^2 f(x, y) dx \right) dy$   
x.  $\int_{-3}^{-\sqrt{5}} \left( \int_0^{\sqrt{9-x^2}} f(x, y) dy \right) dx + \int_{-\sqrt{5}}^0 \left( \int_0^2 f(x, y) dy \right) dx$   
xi.  $\int_0^{\pi/2} \left( \int_0^{3 \cos x} f(x, y) dy \right) dx$
10. i.  $\int_{-2}^0 \left( \int_{-x/2}^1 f(x, y) dy \right) dx + \int_0^1 \left( \int_x^1 f(x, y) dy \right) dx + \int_{-2}^0 \left( \int_{-1}^{x/2} f(x, y) dy \right) dx + \int_0^1 \left( \int_{-1}^{-x} f(x, y) dy \right) dx$   
ii.  $\int_0^4 \left( \int_{\sqrt{x}}^2 f(x, y) dy \right) dx + \int_0^9 \left( \int_{-3}^{-\sqrt{x}} f(x, y) dy \right) dx$
11. i.  $e - 2$   
ii.  $\frac{2a^3}{3}$   
iii.  $\sqrt{5} - \frac{1}{\sqrt{2}} + \frac{1}{2} \log \frac{\sqrt{5}+2}{\sqrt{2}+1} - \frac{1}{6}(5^{3/2} - 2^{3/2})$   
iv.  $\frac{1-\cos 1}{2}$   
v.  $e^4 - 1$   
vi.  $\frac{e-1}{3}$

$$14. \frac{11}{2}$$

$$15. \frac{196}{15}$$

$$16. \frac{1}{4}$$

$$17. \frac{8}{15}$$

$$18. 1$$

$$19. 128\pi$$