CONCENTRATION OF THE INTEGRAL OF IDEMPOTENT EXPONENTIAL POLYNOMIALS

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A problem of some 30 years of history is the question of concentrating the absolute value p-th power integral of idempotent (i.e., 0-1 coefficient) exponential polynomials on arbitrary (small) say open sets. (This occurred and has a relevance in estimating norms of projection operators from L^q spaces to finite dimensional subspaces, spanned by exponentials.) Anderson, Ash, Jones, Rider and Saffari proved in the Comptes Rendus (Paris) in 2000 the possibility of positive concentration for all $p > 1$ and conjectured that concentration already fails for p=1. We disproved this conjecture. Moreover, we even showed that (i) there is concentration for all $p > 0$ (ii) for all $0 < p < 2$, concentration can be derived even for idempotent polynomials having arbitrarily large gaps (iii) for $0 < p < 2$ and also for $2 < p < 4$ the effect of concentration can be as large as is allowed by the obvious fact that absolute value of an idempotent is an even function and hence a set with no mesh with its complement can contain only at most half of the total integral. (iv) even for $p > 4$, the concentration constant can be uniformly bounded from below by at least 0.05. The above results essentially improve all know estimates of the concentration constants for all $p(\neq 2)$. We also explore connections to existing results and conjectures on the Hardy-Littlewood majorant problem. Our work employs several techniques which are of interest themselves.

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