In which domains can one do Fourier Analysis?

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We all know how to do Fourier Analysis in an interval:

with the orthogonal basis

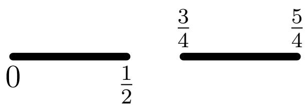
$$e^{2\pi i n x},\ n \in \mathbb{Z}.$$

With a little more effort we can do the same with the domain

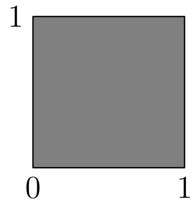
and the orthogonal basis

$$e^{2\pi i 2 n x},\ e^{2\pi i (2n - \frac{1}{2})x},\ n \in \mathbb{Z}.$$

but there is no such orthogonal basis for the domain

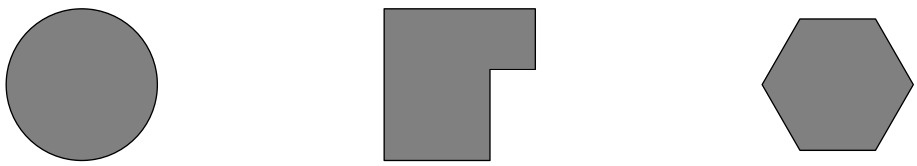


In higher dimension, say dimension 2, we know that the unit square



has the orthogonal basis $e^{2\pi i (m,n)\cdot(x,y)}$, $(m, n) \in \mathbb{Z}^2$.

But what happens with the domains below?



The answer is negative for the disk and affirmative for the other two domains.

Domains that admit such an orthogonal basis of complex exponentials $e^{2\pi i \lambda \cdot x}$, for λ in some set of *frequencies* $\Lambda \subseteq \mathbb{R}^d$, are called *spectral* and the problem of characterizing them has been pursued since the 1970s.

We hope to show some of the rich connections it has with Geometry, Number Theory and Fourier Analysis, and cover some recent results regarding the structure of the spectrum Λ.