THE HILBERT TRANSFORM ALONG FINITE ORDER LACUNARY SETS OF DIRECTIONS

IOANNIS PARISSIS

ABSTRACT. Let $\mathbf{\Theta} \subset S^1$ be a set of directions (where S^1 is the unit circle). For $v \in \mathbf{\Theta}$ we consider the Hilbert transform along the direction given by v

$$H_v f(x) \coloneqq \text{p.v.} \int_{\mathbb{R}} f(x - vt) \frac{dt}{t}, \qquad x \in \mathbb{R}^2, \quad f \in \mathcal{S}(\mathbb{R}^2).$$

This operator is easily seen to be bounded, uniformly in $v \in \mathbf{\Theta}$, by using the boundedness of the Hilbert transform on the real line. Consider now the maximal directional Hilbert transform $H_{\mathbf{\Theta}} f(x) \coloneqq \sup_{v \in \mathbf{\Theta}} |H_v f(x)|$. I will give some background and history relating to boundedness results for $H_{\mathbf{\Theta}}$ and present the following sharp bound: if $\mathbf{\Theta}$ is (a finite subset of) a lacunary set of directions of *finite order* D then

$$\|H_{\mathbf{\Theta}}\|_{p \to p} \eqsim_{p,D} \sqrt{\# \log(\mathbf{\Theta})}, \qquad 1 < p < \infty.$$

I will also discuss some connections between the operator $H_{\mathbf{\Theta}}$ and the Hilbert transform along vector fields and mention some relevant results. This talk reports on recent joint work with Francesco di Plinio (University of Virginia).

IOANNIS PARISSIS.: UNIVERSIDAD DEL PAÍS VASCO (UPV/EHU), DEPARTAMENTO DE MATEMÁTICAS, APARTADO 644, 48080, SPAIN

E-mail address: ioannis.parissis@ehu.es

URL: https://sites.google.com/site/ioannisparissis/home

Date: October 18, 2017.